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Energy-Environmental Efficiency in South America: A Comparative Analysis with DEA under Different Returns to Scale

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Abstract

Given the challenges posed by climate change and the energy transition, assessing the efficiency in the use of environmental and energy resources has become essential to support sustainable public policies. This study analyzes the energy-environmental efficiency of 11 South American countries, considering inputs such as energy consumption, CO_2 emissions and particulate matter (PM2.5), and outputs such as life expectancy and participation of renewable sources. The methodology adopted was Data Envelopment Analysis (DEA), in the CRS (constant returns to scale) and VRS (variable returns to scale) models, with input orientation. A computational tool was also developed in Python, with the PuLP library, to automate the calculations and facilitate replication. The 2023 data were obtained from the Our World in Data database. Preliminary results indicate that Uruguay, Ecuador, Bolivia and Paraguay presented high efficiency in both models, while Venezuela and Chile obtained lower performance. The comparison between the CRS and VRS models highlights the importance of considering the effects of scale to correctly assess efficiency, revealing that part of the inefficiency is related to structural limitations. The findings reinforce the usefulness of DEA as a decision-making tool in sustainability, as well as its potential to guide more equitable and effective energy strategies in South America.

Keywords

Environmental Energy Efficiency, Data Envelopment Analysis (DEA), Sustainability in South America.

1. Introduction

Growing global concerns about climate change, depletion of natural resources and inequality in access to quality energy make it urgent to evaluate sustainable development strategies. Countries at different economic stages face specific challenges in balancing economic growth and environmental responsibility.

South America, for example, has a relatively clean energy matrix in several countries, such as Brazil and Paraguay, thanks to the presence of hydroelectric sources, but it still faces significant difficulties related to pollutant emissions, waste management and low efficiency in converting energy into broad social benefits (World Meteorological Organization, 2023). Given this scenario, assessing the region's energy-environmental efficiency is crucial to identify good practices and promote more effective and sustainable public policies.

The search for analytical methods that integrate economic, environmental and social variables has intensified, and Data Envelopment Analysis (DEA) has emerged as a robust approach for this purpose. Developed by Charnes, Cooper and Rhodes (1978), DEA allows the assessment of the relative efficiency of decision-making units (DMUs), comparing their ability to transform inputs (such as energy consumption and CO_2 emissions) into desirable results (such as Gross Domestic Product - GDP per capita and life expectancy).

Furthermore, the technique enables the identification of benchmarks and good practices, and is widely applied in different sectors, such as health, agriculture, finance, and public infrastructure (Garmatz, Vieira, and Sirena, 2021; Simionato and Cassel, 2019). Recently, its use has been expanded to environmental issues, allowing the integration of ecological and social criteria in the measurement of efficiency, as demonstrated by Zhou et al. (2018), Picazo-Tadeo et al. (2011), and also by more current approaches, such as those proposed by Tran (2025) with the SBM-DEA model and Hadi-Vencheh et al. (2024), which incorporate stochastic analysis and undesirable variables when evaluating the energy sector.

This study aims to apply Data Envelopment Analysis to measure the energy-environmental efficiency from 11 South American countries, using updated data from 2023 from the Our World in Data database. The research differs from others because it uses two distinct DEA models: the CRS (Constant Returns to Scale) model, which assesses global efficiency assuming constant returns to scale, and the VRS (Variable Returns to Scale) model, which allows identifying pure technical efficiency, isolating the effect of the scale of operations (Banker, Charnes & Cooper, 1984). This comparison between the models contributes to a more robust and realistic analysis of relative efficiency among countries with different population sizes, economic structures and levels of sustainable development.

Despite the widespread application of DEA in corporate and institutional contexts, there are still few studies exploring its application in the comparative analysis of sustainable efficiency among South American countries, especially with a simultaneous focus on energy, environmental, and social welfare indicators. Previous studies, such as those by Wanke et al. (2016) and Liu et al. (2020), demonstrate the potential for improving DEA analysis through integration with computational models and automated tools, but there is still a significant gap in the development of accessible and replicable platforms for analyzing environmental efficiency on a continental scale. This research seeks to fill this gap by proposing, in addition to the empirical analysis, a computational tool in Python that automates calculations and facilitates the replication of the model in different contexts and periods.

2. Theoretical Framework

This paper initially discusses energy generation as a central element for socioeconomic development, highlighting the global challenges in the transition from fossil fuel-based energy matrices to more sustainable renewable sources.

Next, we discuss Data Envelopment Analysis (DEA), a nonparametric methodology widely used to measure the relative efficiency of decision-making units. DEA allows performance assessment considering multiple inputs and outputs and is particularly useful for sustainability studies by integrating environmental, social and economic dimensions.

Finally, we explore the application of the DEA-CRS (constant returns to scale) and DEA-VRS (variable returns to scale) models, which allow for complementary analyses: while the first measures global efficiency, the second isolates the effect of operational scale, revealing pure technical efficiency. Understanding these approaches is essential to correctly interpret the results of the empirical analysis proposed in this article.

2.1. Power Generation

Power generation is one of the pillars of economic and social development, being crucial for the functioning of productive activities, urban mobility and the quality of life of the population. However, this process also represents one of the main sources of greenhouse gas emissions, contributing significantly to the worsening of climate change.

In the global context, there is a growing concern about the origin of the energy produced, especially with regard to the transition from fossil to renewable sources, with the aim of promoting a cleaner and more sustainable energy model (World Meteorological Organization, 2023). Recent studies, such as presented by Abbood, Mészáros and Alatawneh (2025), reinforce the need to integrate quantitative approaches to assess the sustainability of transport and energy systems, highlighting the relevance of methods such as DEA in the analysis of environmental efficiency in multiple regions.

In South America, the energy matrix is relatively diversified and, in some cases, heavily based on renewable sources, such as hydroelectric, solar and wind energy. Brazil, for example, has one of the cleanest matrices in the world, with a predominance of hydroelectric energy, although it also faces challenges related to climate dependence and the expansion of other sustainable sources (Simionato & Cassel, 2019). On the other hand, countries such as Venezuela and Argentina still have a significant share of fossil fuels, reflecting a slower energy transition and structural challenges in replacing them with renewable sources (Our World in Data, 2024).

In addition to the composition of the power matrix, it is essential to analyze the efficiency with which countries convert energy into economic development and quality of life. In this sense, authors such as Picazo-Tadeo et al. (2011) and Garmatz, Vieira and Sirena (2021) highlight the importance of using composite indicators and quantitative approaches, such as Data Envelopment Analysis (DEA), to measure energy performance in an integrated manner with environmental and social aspects.

New approaches, such as the one proposed by Ji, Wei and Ma (2024), with the incremental DEA model applied to sustainability, further expand the possibilities of analysis by considering the variation in efficiency over time and dynamic scenarios. This approach allows us to assess not only the volume of energy generated, but also the sustainability of the production process and its contribution to the well-being of the population.

Therefore, understanding the power generation patterns of South American countries and their relationship with sustainable development is essential to guide public policies, investments in energy infrastructure and regional strategies to mitigate climate change. This analysis becomes even more relevant in a scenario of international pressure for decarbonization, and commitments made in multilateral agreements on climate and energy.

2.2. Data Envelopment Analysis (DEA)

DEA is a quantitative technique used to assess the relative efficiency of a set of decision-making units, such as organizations and programs. It builds an efficiency frontier with the most efficient units in transforming certain inputs into certain products (GARMATZ, VIEIRA & SIRENA 2021). The technique also has the advantage of being able to identify good practices, as it highlights the most efficient units, which can be used as a benchmark to improve the performance of others.

In this context, DEA makes performance comparisons between Decision Making Units (DMUs), which perform similar processes, based on the relationship between inputs and the results of each unit, called outputs (SIMIONATO and CASSEL, 2019).

The DEA method has been successfully applied to the study of the efficiency of public administration and non-profit organizations. It has been used to compare educational departments (schools, colleges, universities and research institutes), health establishments (hospitals, clinics), prisons, agricultural production, financial institutions, countries, armed forces, sports, transportation (road maintenance, airports), restaurant chains, franchises, courts of justice, cultural institutions (theater companies, symphony orchestras) among others (GARMATZ, VIEIRA & SIRENA 2021).

Besides the traditional applications, DEA has been used to assess efficiency in complex and dynamic contexts, such as natural resource management and sustainability. For example, Zhou et al. (2018) applied the method to measure the energy efficiency of Chinese regions, incorporating environmental variables such as CO2 emissions. Similarly, Picazo-Tadeo et al. (2011) used DEA to analyze the performance of organic farms in Spain, demonstrating how the technique can integrate economic and ecological criteria. These studies highlight the flexibility of DEA in adapting to different objectives, from maximizing outputs to minimizing negative impacts.

Another significant advance is the combination of DEA with other methodologies, such as regression analysis or machine learning, to identify determinants of efficiency. Wanke et al. (2016) used DEA followed by logistic regression models to investigate factors that influence port efficiency in Brazil, such as infrastructure and geographic location. Liu et al. (2020) integrated DEA with neural networks to predict the efficiency of public hospitals in the US, highlighting how the technique can be enhanced with predictive approaches. These combinations expand the analytical potential of DEA, allowing not only to diagnose inefficiencies but also to propose solutions based on identified patterns.

2.3. DEA-CRS versus DEA-VRS Model

The DEA-CRS (Constant Returns to Scale) model, introduced by Charnes, Cooper and Rhodes (1978), assumes that all DMUs (Decision Making Units) operate at an optimal scale, where proportional changes in inputs result in proportional changes in outputs. However, this assumption is often criticized for ignoring real situations where organizations present increasing returns (economies of scale) or decreasing returns (diseconomies of scale).

To overcome this limitation, Banker, Charnes, and Cooper (1984) proposed the DEA-VRS (Variable Returns to Scale) model, which relaxes the proportionality constraint and allows technical efficiency to be assessed regardless of unit size. This model is especially relevant in sectors with high operational heterogeneity and is widely used by researchers using Data Envelopment Analysis. A recent example is the work of Yin, Alnafrah, and Zhou (2024), who developed a DEA model with network bias correction to assess sustainable practices and efficient resource management across OECD countries, highlighting the advantages of the VRS model in contexts with significant structural variations.

Studies as developed by Marinho and Façanha (2003) applied DEA-VRS in Brazilian hospitals and found that smaller units, such as health centers, often have greater technical efficiency due to less bureaucracy, while large hospitals suffer from diseconomies of scale. In agriculture, research such as that by Alves et al. (2015) used DEA-VRS to compare rural properties in the Cerrado region of Brazil and found that medium-sized farms (between 100 and 500 hectares) were the most efficient, benefiting from scale without losing managerial agility.

Simply put, the CRS model is used when one wants to assume that production is proportional to inputs, regardless of the size of the unit being analyzed, and is ideal for situations in which scale does not affect efficiency — a scenario that is more common in theoretical models. The VRS model is more appropriate when there are increasing or decreasing returns to scale, capturing pure technical efficiency, and is common in practical applications where units operate at different sizes and with different operational structures.

3. Data and Methodological Procedures

This study uses Data Envelopment Analysis (DEA) to assess the energy-environmental efficiency from 11 South American countries. In addition to applying the models, a comparison will also be made between the results obtained by the CRS (Constant Returns to Scale) and VRS (Variable Returns to Scale) models, enabling a more comprehensive analysis of efficiency.

The CRS model allows us to observe the overall efficiency of the countries, while the VRS model isolates the effect of scale, allowing us to assess pure technical efficiency. To operationalize the analysis, a tool was developed in Python using the *PuLP library*, which transforms the DEA models into linear programming problems, solving them computationally in an automated manner.

3.1. Research Data

Data Envelopment Analysis requires the clear definition of Decision-Making Units (DMUs) inputs (resources consumed) and outputs (results generated). The choice of parameters reflects the climate urgency highlighted by the World Meteorological Organization in its general climate outlook published in 2023, with the focal object of this work being energy-environmental efficiency through the following parameters:

- Inputs (indicators of environmental pressure and consumption):

    1. Energy consumption per capita (kWh/inhab.): Measures energy intensity.

    2. CO_2 emissions per capita (tons/inhab.): Represents the direct climate impact.

    3. Particulate matter (PM2.5) from burning (millions of tons: Indicates local environmental degradation.

- Outputs (socioeconomic benefits and sustainability):

    1. GDP per capita (US$): Reflects economic development.

    2. Share of renewables in the electricity matrix (%): Measures energy transition.

    3. Life expectancy at birth (years): Captures social well-being.

The countries chosen were Brazil, Argentina, Uruguay, Chile, Peru, Bolivia, Paraguay, Colombia, Suriname, Venezuela and Ecuador. The information regarding the parameters collected for each of the countries in 2023 is available in Table 1.

This study analyzes 11 South American countries (DMUs) in 2023, selected by complete data availability in the Our World in Data (OWID) database, which consolidates official sources such as the World Bank, WHO, IPCC and UN (updated until January 2025).

3.2. Application of DEA through the tool

The implementation of Data Envelopment Analysis (DEA) was carried out using a computational tool developed in Python, with the PuLP library for modeling and solving linear programming problems. This approach allows calculating the relative efficiency of decision-making units (DMUs) considering multiple inputs (resources) and outputs (products or results), with flexibility to adjust parameters according to the analysis needs.

The tool developed allows data entry in two ways: manual insertion or import via CSV file, following a standardized format with columns named as input_1, input_2, ..., output_1, output_2, etc. To facilitate the interpretation of the results, a configuration file (config.txt) was created, which applies descriptive labels to the variables and DMUs (Decision Making Units), ensuring greater clarity in the analysis.

In the model configuration step, the user can choose between two main approaches: constant returns to scale (CRS) or variable returns to scale (VRS), depending on the analysis need. The tool was designed with flexibility to incorporate other DEA models in future updates, since its code is open source and allows adaptations according to demand.

Table 1. Data used for calculations of efficiency frontiers

	Inputs			Outputs		
Country	**Power consumption per inhabitant (kwh)**	**CO2 emissions per inhabitant (tons)**	**Pm2,5 from wildfired (million tons)**	**GDP per capita 2023 ($)**	**Participation of electricity generated by renewable energy (%)**	**Life expectancy at birth (years)**
Brazil	17.806	2,3	2,3	19.019	88,68	75,8
Argentina	22.278	4,3	0,209015	27.105	34,70	77,4
Uruguay	18.317	2,3	0,002379	31.019	89,77	78,1
Chile	25.679	3,9	0,106273	29.463	60,82	81,2
Peru	10.009	1,7	0,096242	15.294	58,87	77,7
Bolivia	7.062	1,9	0,660359	9.844	28,81	61,4
Paraguay	15.782	1,2	0,149700	15.783	100,0	73,8
Colombia	12.028	2,0	0,115652	18.692	66,49	77,7
Suriname	20.285	4,2	0,006944	19.044	47,10	73,6
Venezuela	24.383	3,5	0,266961	8.559	76,50	72,5
Ecuador	12.881	2,4	0,032237	14.472	80,83	77,4

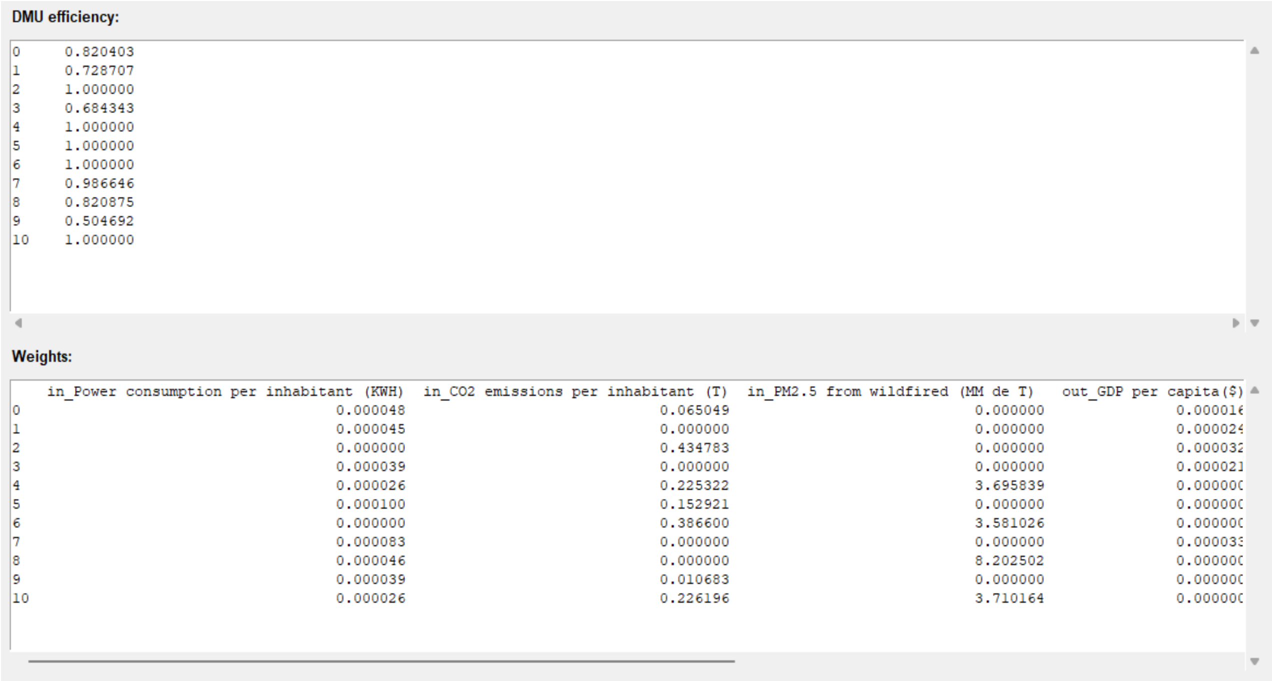
Source: Authors (2025)

After configuration, the tool performs calculations by solving the linear optimization problem for each DMU, determining its relative efficiency on a scale from 0 to 1 — with 1 indicating total efficiency. The results are presented in a graphical interface organized (Figure 1) into two main parts:

- DMU Efficiency Section: Displays the efficiency score of each unit, available for viewing;
- Optimal Weights Section: Details the relative contribution of each input and output to the efficiency calculation.

The tool implements the classical DEA model (CCR for constant returns to scale - CRS and BCC for variable returns - VRS) through linear programming equations. To perform all calculations and structure the program, several libraries were used, available for viewing in Table 2.

Figure 1. Data displayed in the graphical interface of the generated tool



DMU efficiency:

0	0.820403
1	0.728707
2	1.000000
3	0.684343
4	1.000000
5	1.000000
6	1.000000
7	0.986646
8	0.820875
9	0.504692
10	1.000000

Weights:

	in_Power consumption per inhabitant (KWH)	in_CO2 emissions per inhabitant (T)	in_PM2.5 from wildfired (MM de T)	out_GDP per capita($)
0	0.000048	0.065049	0.000000	0.000016
1	0.000045	0.000000	0.000000	0.000024
2	0.000000	0.434783	0.000000	0.000032
3	0.000039	0.000000	0.000000	0.000021
4	0.000026	0.225322	3.695839	0.000000
5	0.000100	0.152921	0.000000	0.000000
6	0.000000	0.386600	3.581026	0.000000
7	0.000083	0.000000	0.000000	0.000033
8	0.000046	0.000000	8.202502	0.000000
9	0.000039	0.010683	0.000000	0.000000
10	0.000026	0.226196	3.710164	0.000000

Source: Authors (2025)

The library responsible for solving the calculations is Pulp, which implements the equations of the CRS models of Charnes, Cooper and Rhodes (CCR - 1978) and the VRS of Banker, Charnes and Cooper (BCC - 1984), based on the linear programming theory of Dantzig (1951).

Table 2. Libraries used in building the tool

Library	Motivation
Pandas	Efficiency in handling tabular data and integration with CSV files.
Pulp	Simplicity in modeling linear problems and compatibility with free solvers.
Tkinter	Python standard library for graphical interfaces, without external dependencies.
Os and Re	Native Python features for auxiliary tasks (paths, strings).

Source: Authors (2025)

For the CCR – CRS model, Equation 1 is the main one, being subject to Equations 2 (normalization restriction), Equation 3 (efficiency frontier) and Equation 4 (non-negativity).

$$Maximize\ \theta = \sum_{r=1}^{s} u_r y_{r0} \tag{1}$$

$$\sum_{i=1}^{m} v_i \cdot x_{i0} = 1 \tag{2}$$

$$\sum_{r=1}^{s} u_r \cdot y_{rj} - \sum_{i=1}^{m} v_i \cdot x_{ij} \leq 0 \qquad \forall j \tag{3}$$

$$u_r, v_i \geq 0 \qquad \forall r, i \tag{4}$$

VRS, on the other hand, uses Equation 5 as the main one, being subject to Equations 6, 7 and 8.

$$Maximize\ \theta = \sum_{r=1}^{s} u_r y_{r0} + \mu_0 \tag{5}$$

$$\sum_{i=1}^{m} v_i \cdot x_{i0} = 1 \tag{6}$$

$$\sum_{r=1}^{s} u_r \cdot y_{rj} - \sum_{i=1}^{m} v_i \cdot x_{ij} + \mu_0 \leq 0 \qquad \forall j \tag{7}$$

$$u_r, v_i \geq 0 \qquad \forall r, i \tag{8}$$

Being:

x_i: Quantity of input *i* used by decision unit *j*

x_{i0}: Quantity of input *i* used by the evaluated decision unit (DMU_0)

v_i: Weight assigned to input *i*, determined by the model

y_{rj}: Amount of output *r* produced by the decision unit *j*

u_r: Weight assigned to output *r*, determined by the model

i = 1, 2, ..., m: Index of input variables (*m* inputs)

r = 1, 2, ..., s: Index of output variables (*s* outputs)

j: Decision unit index (DMUs)

μ_0 = Free intercept term that captures scale effects on the production frontier

4. Results and Discussions

The results derived from the application of Data Envelopment Analysis (DEA) provided a comprehensive assessment of the energy-environmental efficiency of 11 South American countries, based on indicators such as energy consumption, CO_2 emissions, PM2.5 levels, renewable energy share, GDP per capita, and life expectancy. By employing both the CRS (Constant Returns to Scale) and VRS (Variable Returns to Scale) models, the study enabled a comparative evaluation of overall efficiency and pure technical efficiency, revealing not only the countries operating on the efficiency frontier but also those whose performance was constrained by scale-related limitations. The following section presents the empirical findings of both models, highlighting the most efficient countries, identifying key influencing variables, and discussing the implications of these results for public policy and sustainable development strategies in the region.

Table 3 presents the results of the DEA-CRS model applied to the energy-environmental efficiency of 11 South American countries. The model considers constant returns to scale and allows us to identify which countries most efficiently use their energy and environmental resources to generate social well-being. The countries that achieved maximum efficiency (EF = 1.000)—Uruguay, Peru, Bolivia, Paraguay, and Ecuador—are considered benchmarks for the others. On the other hand, countries such as Venezuela (EF = 0.505) and Chile (EF = 0.684) had the lowest efficiency indices, indicating significant opportunities for improvement.

Table 3. Efficiency and weight of each variable in the DEA-CRS model

Country	EF	Power	CO_2	PM2.5	GDP	Renewable Sources	Life Expectancy
Brazil	0.820	0.000048	0.065049	0.000000	0.000016	0.005717	0.000000
Argentina	0.729	0.000045	0.000000	0.000000	0.000024	0.000000	0.001120
Uruguay	1.000	0.000000	0.434783	0.000000	0.000032	0.000000	0.000000
Chile	0.684	0.000039	0.000000	0.000000	0.000021	0.000000	0.000972
Peru	1.000	0.000026	0.225322	3.695839	0.000000	0.000000	0.012870
Bolivia	1.000	0.000100	0.152921	0.000000	0.000000	0.000000	0.016287
Paraguay	1.000	0.000000	0.386600	3.581026	0.000000	0.010000	0.000000
Colombia	0.987	0.000083	0.000000	0.000000	0.000033	0.005564	0.000000
Suriname	0.821	0.000046	0.000000	8.202502	0.000000	0.000000	0.011153
Venezuela	0.505	0.000039	0.010683	0.000000	0.000000	0.005519	0.001138
Ecuador	1.000	0.000026	0.226196	3.710164	0.000000	0.000000	0.012920

Source: Authors (2025)

The weights assigned to each variable reveal the relative importance of the indicators in each country's performance. In cases like Uruguay, the most relevant variable was CO_2 emissions, while in Peru, the highlight was particulate matter (PM2.5) control, associated with a good life expectancy. Brazil, with an efficiency score of 0.820, was more strongly influenced by variables related to the energy matrix, such as renewables and emissions, with zero weight assigned to PM2.5 and life expectancy, indicating a lower impact of these variables on its score.

The analysis shows that different efficiency trajectories are associated with different resource use structures and socio-environmental conditions. The DEA-CRS model allows us to identify the most efficient countries, but it also highlights the need for specific strategies for each national context. The zero weighting of some variables in inefficient countries reinforces the importance of policies targeting these neglected dimensions, making the analysis a useful tool for guiding sustainability actions and investment allocation.

Table 4. Efficiency and weight of each variable in the DEA-VRS model

Country	EF	Power	CO_2	PM2.5	GDP	Renewable sources	Life Expectancy
Brazil	0.846	0.000056	0.000000	0.000000	0.000014	0.007899	0.001942
Argentina	0.729	0.000045	0.000000	0.000000	0.000024	0.000000	0.000188
Uruguay	1.000	0.000054	0.000000	2.603134	0.000013	0.000000	0.000000
Chile	1.000	0.000039	0.000000	0.000000	0.000018	0.000000	0.101513
Peru	1.000	0.000025	0.235598	3.677498	0.000000	0.000000	0.000000
Bolivia	1.000	0.000033	0.389528	0.036384	0.000000	0.000000	0.000000
Paraguay	1.000	0.000000	0.431450	3.221506	0.000000	0.000000	0.000000
Colombia	0.992	0.000083	0.000000	0.000000	0.000021	0.011694	0.002875
Suriname	0.870	0.000046	0.000000	8.448641	0.000000	0.000000	0.000000
Venezuela	0.509	0.000040	0.008650	0.000000	0.000000	0.005477	0.000000
Ecuador	1.000	0.000025	0.235598	3.677498	0.000000	0.000000	0.000000

Source: Authors (2025)

Table 4 presents the results of the DEA-VRS (Variable Returns to Scale) model, which measures pure technical efficiency by removing the influence of operational scale. In contrast to the CRS model, the VRS approach identified six countries (Uruguay, Chile, Peru, Bolivia, Paraguay, and Ecuador), as fully efficient (EF = 1.000), indicating their capacity to convert energy and environmental inputs into desirable outputs regardless of scale constraints. This suggests that these countries adopted strategies or structural conditions that allowed them to optimize their resource use within their own operational contexts. Notably, Brazil and Argentina, which were inefficient under the CRS model, showed improved scores under the VRS framework, though without reaching full efficiency. This reinforces the importance of distinguishing technical inefficiency from inefficiency caused by scale limitations, especially in regional contexts marked by institutional and economic heterogeneity.

The weight distribution across variables reveals distinct patterns among the countries. In Brazil, for example, the efficiency score was primarily influenced by renewable energy use and life expectancy, while variables like CO_2 emissions and GDP per capita had negligible impact. For the efficient countries, the relevant drivers varied: in Paraguay and Peru, high weights were assigned to PM2.5 reduction and CO_2 performance, while Chile's efficiency was heavily supported by life expectancy. Venezuela remained the least efficient country (EF = 0.509), despite attributing weight to renewable energy and life expectancy, indicating a disconnect between resource use and outcome generation. These findings suggest the existence of multiple efficiency frontiers and highlight the relevance of country-specific strategies when addressing sustainable development. The VRS model thus offers a more nuanced and equitable lens for evaluating national energy-environmental performance in structurally diverse regions like South America.

The comparison between the CRS and VRS models reveals important distinctions regarding how efficiency is interpreted in the context of heterogeneous countries. The CRS model, by assuming constant returns to scale, evaluates overall efficiency and implicitly requires that all countries operate at an optimal scale. Under this assumption, only five countries Uruguay, Peru, Bolivia, Paraguay, and Ecuador achieved full efficiency (EF = 1.000), while others, such as Venezuela and Chile, registered significantly lower scores. This suggests that the CRS model tends to penalize countries whose structural conditions, economic scale, or institutional capacity deviate from the assumed optimal frontier, regardless of how well they manage their resources within those constraints.

In contrast, the VRS model isolates pure technical efficiency by allowing for variable returns to scale, providing a more flexible and realistic assessment of performance. As a result, the number of fully efficient countries increased to six Uruguay, Chile, Peru, Bolivia, Paraguay, and Ecuador, indicating that these nations made the best possible use of their inputs within their operational contexts, even if not at optimal scale. Additionally, countries like Brazil and Argentina, which were inefficient under CRS, showed better performance under VRS, highlighting that their inefficiencies are likely due to scale-related factors rather than mismanagement of resources. This comparison underscores the importance of employing both models in empirical studies: while CRS identifies global efficiency, VRS offers a more equitable perspective by accounting for structural asymmetries, making it particularly relevant in regional analyses marked by economic and institutional disparities.

This distinction between models is crucial for formulating public policies and development strategies. By identifying whether the inefficiency is technical or scale-related, efforts can be targeted more precisely: if the problem is resource misallocation, the solution may lie in improving management and optimizing processes; if it's scale-related, structural interventions or regional partnerships may be more appropriate.

5. Final Remarks

This study assessed the energy-environmental efficiency of 11 South American countries using Data Envelopment Analysis (DEA) under two methodological approaches: Constant Returns to Scale (CRS) and Variable Returns to Scale (VRS). The analysis incorporated key indicators related to energy consumption, carbon emissions, air quality (PM2.5), renewable energy share, GDP per capita, and life expectancy, offering a multidimensional view of sustainability performance. By comparing CRS and VRS results, the study revealed how structural and scale-related constraints influence national efficiency levels, providing a nuanced understanding of sustainability dynamics in the region.

The CRS model, which measures global efficiency, identified five countries—Uruguay, Peru, Bolivia, Paraguay, and Ecuador—as benchmarks, while others exhibited varying degrees of inefficiency. However, the VRS model, which isolates pure technical efficiency by accounting for scale heterogeneity, classified six countries as fully efficient, including Chile. This distinction highlights the importance of using both models in parallel: the CRS model offers insights into overall performance, while the VRS model reveals which countries are optimizing their resources given their existing structural limitations.

The findings demonstrate that efficiency is not determined solely by the volume of resources available, but by how effectively these are transformed into social and environmental outcomes. Countries with modest energy consumption or lower GDP per capita—such as Bolivia and Paraguay, were able to achieve full efficiency by leveraging renewable energy and minimizing environmental degradation. Conversely, some countries with higher economic indicators underperformed due to inefficiencies in environmental management or weak social outcomes. These results underscore the need for targeted, context-sensitive policies that prioritize integrated development and sustainability.

In addition to the empirical contributions, this study also developed an open-source computational tool in Python that automates DEA modeling and supports replication in different contexts. This innovation enhances the applicability of DEA in environmental and energy policy analysis, offering researchers and policymakers a practical instrument for evidence-based decision-making. Future research could extend the model by incorporating temporal data to capture efficiency trends over time or by including governance and institutional quality variables to deepen the understanding of sustainability drivers in South America.

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Conflict of Interest Declaration

The authors have no conflicts of interest to declare. All co-authors have seen and agree with the contents of the manuscript and there is no financial interest to report.

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