



ONLINE INSTRUMENTAL ORCHESTRATION: PERSPECTIVES FOR THE INITIAL EDUCATION OF PRE-SERVICE MATHEMATICS TEACHERS IN THE CONTEXT OF THE SUPERVISED TEACHING PRACTICE

Orquestração instrumental on-line: perspectivas para a formação inicial de futuros professores de matemática no contexto do estágio curricular supervisionado

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Abstract

The role of the supervised teaching practice in initial teacher education has been the focus of investigations in the area of mathematics education due to the emergence, in this scenario, of theoretical and practical knowledge for the professional performance of pre-service mathematics teachers. In this article, we aim to discuss the use of the notion of (online) instrumental orchestration as a theoretical-methodological proposal for the development of classes in the context of a supervised teaching practice. For this purpose, we present the components of instrumental orchestration, linking the theoretical and practical dimensions, to illustrate the data produced from the implementation of orchestrations by a pre-service mathematics teacher under the supervision of a teacher-supervisor, in an online class with students in the 6th grade of elementary school at a federal public school. Among the results, we highlight the didactical configurations, the exploitation modes, and the didactical performances around using a digital artifact which made it possible to perform interactive simulations for the study of the equality relations, through collective work, in the remote teaching modality, between the participants of the research. In short, we found the importance of systematically planning the arrangement of the teaching environment for online classes in the context of supervised teaching practice.

Keywords: Teacher education; Supervised teaching practice; Online instrumental orchestration; Digital technologies in mathematics education; Equality relations.

Resumo

O papel do Estágio Curricular Supervisionado na formação inicial docente tem sido foco de investigações na área de Educação Matemática devido à emergência, neste cenário, dos conhecimentos teórico-práticos para a atuação profissional dos futuros professores de Matemática. Neste artigo, objetivamos discutir o uso da noção de Orquestração Instrumental (On-line) como proposta teórico-metodológica para o desenvolvimento de aulas no contexto do estágio curricular supervisionado. Para tanto, apresentamos os componentes da Orquestração Instrumental, atrelando as dimensões teórica e prática, a fim de ilustrar os dados produzidos a partir da implementação de orquestrações, por parte de um estagiário de Matemática sob a supervisão de um professor, em uma aula on-line com alunos do 6º ano do ensino fundamental de uma escola pública federal. Dentre os resultados, destacamos as configurações didáticas, os modos de execução e as performances didáticas em torno da utilização de um artefato digital que permitiu realizar simulações interativas para o estudo das relações de igualdade, mediante o trabalho coletivo, na modalidade do ensino remoto, com os participantes da pesquisa. Em suma, constatamos a importância de planejar sistematicamente o arranjo do ambiente de ensino para as aulas on-line no contexto do estágio curricular supervisionado.

Palavras-Chave: Formação de Professores; Estágio Curricular Supervisionado; Orquestração Instrumental On-Line; Tecnologias Digitais em Educação Matemática; Relações de Igualdade.

INTRODUCTION

Teacher education, whether initial or continuing, is a relevant event in teachers' professional trajectory, given that the professional knowledge necessary for teaching work is developed during the formative process (ASSIS; GITIRANA; TROUCHE, 2019). In this sense, as Trivilin and Ribeiro (2015) point out, it is essential to think of new methodological approaches in teacher education to articulate teaching knowledge. In the context of initial education, some investigations in the area of mathematics education highlight the role of the supervised teaching practice for the professional development of pre-service mathematics teachers (PINA; BRAGA; FIORENTINI, 2021; ASSIS; GITIRANA; TROUCHE, 2019; IGNÁCIO, 2018; ZIMMER, 2017; TEIXEIRA; CYRINO, 2015), since, in this scenario, theoretical-practical knowledge emerges.

Given this understanding, in this article, we approach a discussion on how the notion of online instrumental orchestration – OIO (GITIRANA; LUCENA, 2021), adapted from the instrumental orchestration – IO model (TROUCHE, 2004a; DRIJVERS *et al.*, 2010), can methodologically guide the educational practices of mathematics by using digital artifacts, within the scope of the supervised teaching practice¹, through collaborative work involving a

¹As part of the project “Do cálculo do termo desconhecido à equação do 1º grau: recursos para o estudo de relações e propriedades de igualdade no Ensino Fundamental” [From the calculation of the unknown term to the 1st-degree equation: resources for the study of relations and properties of equality in Elementary School], within the scope of the research group Scientific Laboratory of Learning, Research and Teaching (LACAPE), funded by the Pernambuco State Foundation for Science and Technology (FACEPE) – Announcements 01/2020 and 01/2021.

pre-service teacher, a teacher-supervisor and a lecturer-advisor, based on the following assumption:

The collaborative work between the in-service teacher and the pre-service teacher is fundamental. Assuming the school as a privileged locus of education means replacing a strongly individualistic culture with a collaborative culture, considering its systemic character (CYRINO; GRANDO, 2022, p.10).

Based on the perspective of collaboration among pre-service teachers and their supervisors, we aim to discuss the use of the notion of online instrumental orchestration as a theoretical-methodological proposal for the development of classes in the context of the supervised teaching practice. Thus, we refined the aspects of the theoretical framework of (online) instrumental orchestration, before weaving the methodological aspects of the research study presented in this article, the analysis of the results, and the considerations about the limitations and perspectives for future investigations.

(ONLINE) INSTRUMENTAL ORCHESTRATION

Before entering into the conceptualization of the online instrumental orchestration – OIO, we restate some considerations related to the conception of instrumental orchestration – IO, to clarify the previous theoretical background of that model.

Originally, the IO model advocated the use of artifacts in the educational field, in environments rich in technologies, in particular digital ones, to contribute to the mathematics teaching and learning processes (LUCENA; GITIRANA; TROUCHE, 2019). In the definition proposed by Trouche (2004a):

An instrumental orchestration is the systematic and intentional arrangement of the elements (artifacts and human beings) of an environment, conducted by an agent (teacher) to accomplish a given situation and, in general, to guide the learners in the instrumental genesis and evolution and balance of their systems of instruments (TROUCHE, 2004a, p. 126).

By adopting the metaphor of orchestration, Trouche (2004b) equates the teacher with a conductor, the students with musicians, the classroom with an orchestra, technologies with musical instruments, and teaching environments with repertoires.

The concepts of the instrumental approach have naturally led to a reflection on the teacher's role not as a participant in the orchestra, but as a conductor of an orchestra, responsible for the set of instruments in the class. The class is then seen as an orchestra playing a mathematical score (BELLEMAIN; TROUCHE, 2019, p. 110).

For the development of the notion of IO, Trouche (2004a, 2004b) takes as a basis the two processes involved in an *instrumental genesis* (instrumentation and instrumentalization), woven into the *instrumental approach* (RABARDEL, 1995). In the *instrumentation process*, the artifact intervenes in the development of the subject's thinking and the *instrumentalization process*, the subject adapts the artifact to a certain intentionality in a given situation.

We emphasize that IO consists of three components: *didactical configuration*, *exploitation mode* (TROUCHE, 2004a; 2004b) and *didactical performance* (DRIJVERS *et al.*, 2010). Based on Lucena, Gitirana, and Trouche (2016, p. 3), we have that:

The didactical configuration concerns the organization of the classroom and the teacher's didactical choices regarding the mathematical task, the resources to be made available, and the functions of the individuals involved, among other aspects. The exploitation mode, on the other hand, consists of the operationalization of the didactical configuration previously developed by the teacher, focusing on the students' instrumental genesis. In the didactical performance, the teacher, based on the exploitation mode, can perceive situations that were not foreseen in the didactical configuration. Faced with these, the teacher makes ad hoc decisions emerge. In addition, he/she can analyze whether the performance of the instrumental orchestration created was favourable or not to their didactic objectives.

The *didactical configuration* and the *exploitation mode* are essential for teaching planning, since the teacher gets organized in advance, based on the formative and learning objectives (for example: solving problems on the theme *equality relations* using digital artifacts) to propose a didactic scenario conducive to the development of students' instrumental genesis. However, when exploring the didactical scenario, many aspects of the predicted IO appear, and other new ones emerge, since what is now at stake are not only the teachers' expectations but the reality of the classroom full of students' subjectivities. Therefore, when the teacher analyzes the *didactical performance* of the IO executed, he/she can evaluate both the points of their teaching work – planning and teaching – and student learning.

In addition, it is in the *didactical performance* that two didactic phenomena to be analyzed take place: *ad hoc decisions* (DRIJVERS *et al.*, 2010) and *ad hoc reactions* (LUCENA; GITIRANA; TROUCHE, 2019). During the *didactical performance*, the teacher faces unexpected situations. This demands unplanned decisions, which Drijvers *et al.* (2010) call *ad hoc decisions*, i.e., those of a didactic-pedagogical nature, that have objectives and seek to ensure that the orchestra performs well. On the other hand, students' *ad hoc reactions* do not have a didactic-pedagogical perspective, because the purpose is to solve the situations proposed by the teacher or to perform well in the developing orchestra (LUCENA; GITIRANA; TROUCHE, 2019).

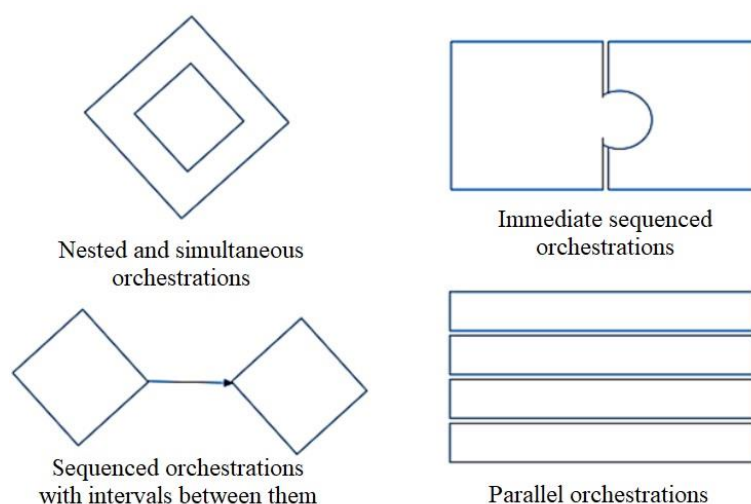
Based on Rousson (2017), we can distinguish two major IO categories, due to the activity proposed to students: the *collective IO* and the *individual IO*. In the *collective IO*, students have time to discuss the task with each other and/or the teacher. In the *individual IO*, students work individually, in pairs or in groups to solve the task.

According to Rousson (2017), among the possible *collective IO*, there are the following types: *technical-demo*; *relation of different supports*; *teachers' explanation*; *teachers' guidance and explanation*; *discussion between stakeholders*; and *sherpa*. Regarding *individual IO*, there is the *work and follow-up* type.

In this work, we use the *technical-demo* and *discussion between stakeholders*. The *technical-demon* type of the collective IO concerns the teacher's presentation of artifact techniques (DRIJVERS *et al.*, 2010). For example, when the teacher shows how to use a calculator, a compass, a ruler, and an interactive simulation, GeoGebra, among others. In the case of digital artifacts, it is important to emphasize that the technical demonstration refers to the presentation of its tools and functionalities. In the *discussion between stakeholders* type of collective IO, the teacher does not limit so much the moments of discussion that aim to focus on understanding the tasks and enhancing the students' collective instrumental genesis (ROUSSON, 2017).

Questioning the possibility of proposing more than one orchestration in a class, Gitirana and Lucena (2021) adapted the IO composition models, inaugurated by Lucena (2018), into: (i) *nested and simultaneous*, (ii) *immediate sequenced*, (iii) *sequenced with intervals between them*, and (iv) *parallel*.

Figure 1 – Models of compositions of instrumental orchestrations



Source: Gitirana and Lucena (2020, p. 4, adapted from Lucena, 2018, translated by the authors).

As shown in figure 1, the *composition of nested and simultaneous orchestrations* refers to the different IOs that are executed simultaneously. In this type, those IOs are mutually dependent because for one to occur, the other must also occur. In the *composition of sequenced orchestrations*, we have different IOs executed one after the other or with some time (hours, days, weeks, etc.) between them. Finally, we have the *composition of parallel orchestrations*, which are different and performed independently and simultaneously.

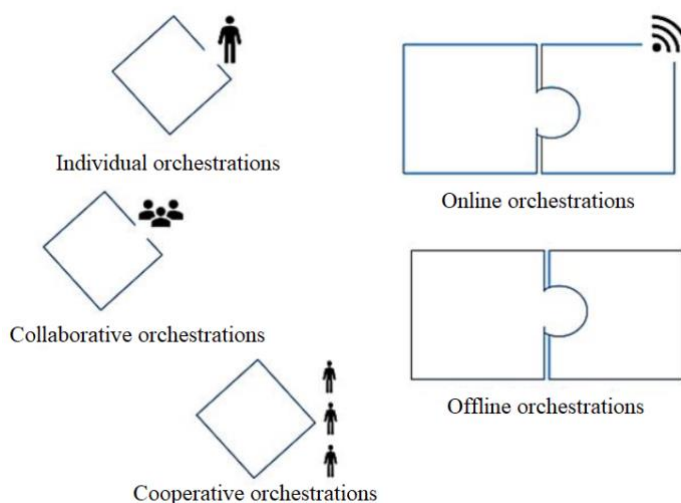
Instrumental orchestration compositions refer to the structuring of orchestrations that occur together, i.e., they are interconnected so that the teacher's didactic intention that the students learn specific mathematical knowledge is achieved. Those compositions do not imply claiming that they are dependent on each other, given that each has the three moments of an orchestration (didactical configuration, exploitation mode, and didactical performance), but rather that, when composed, they allow more meanings to the teaching and learning processes.

Specifically, taking into account the context of remote teaching, Gitirana and Lucena (2021), supported by the definition of IO proposed by Trouche (2004b), put forward the following notion of *online instrumental orchestration*:

An online instrumental orchestration (online IO) is the systematic and intentional arrangement of the elements (artifacts, human beings, and time) of an environment formed by *different, all-connected, geographic and virtual spaces* conducted by agents (teacher(s) and monitor(s)) to accomplish a given situation and, in general, guide their apprentices *remotely, synchronously and/or asynchronously* in their instrumental genesis and the evolution and balance of their systems of instruments (GITIRANA; LUCENA, p. 376, emphasis added).

The italics in this definition are justified by the aspects to be considered when preparing an online IO. Thus, as illustrated below (figure 2), the remote IO model was developed for the remote teaching modality, highlighting online and offline ways to execute an IO, synchronous and asynchronous class formats, and the individual, collaborative or cooperative forms of work that can be carried out by the subjects involved in this IO (GITIRANA; LUCENA, 2021).

Figure 2 – Types of remote instrumental orchestrations and working modes



Source: Gitirana and Lucena (2020, p. 5, translated by the authors).

As shown in figure 2, we have two types of remote IO: *online* and *offline*. According to Gitirana and Lucena (2021), *online IO* can be developed in real time or at different times, depending on the situation at stake. However, it needs internet. While the *offline IO* does not require the internet; it must be performed based on the teacher's propositions and guidelines. Moreover, about the way of working that students can adopt, we have the following: individual – each one works alone; collaborative – everyone working together focused on a common goal, although they play different roles; and cooperative – the participants are divided to carry out the work.

A substantial element, in the context of online teaching, discussed by Gitirana and Lucena (2021), is the constitution of the physical space of the online IO, which, in the teaching modality mentioned above, the geography of space is no longer physical (that is, in person), but virtual. Thus, several particular aspects must be considered in online teaching, namely: the sociocultural context, the socioeconomic context, the context of emerging digital and non-digital artifacts and connection conditions (GITIRANA; LUCENA, 2021).

As for the pedagogical practice, we must take into account the particularities of the online teaching modality, as the teacher does not have free access to the same spaces and artifacts, etc., of the face-to-face teaching. For example, in the face-to-face modality, the teacher can use manipulative materials to develop an instrumental orchestration. In online classes, the teacher offers several digital artifacts. In this way, before the class, they must develop and share

with students a *guidance protocol* (GITIRANA; LUCENA, 2021) containing the resources required for their participation in the class, and the organization and conduct to be adopted to plan and access the virtual environment. That is, it is fundamental to clarify in advance the *didactical configuration* and the *exploitation mode* of the online IO.

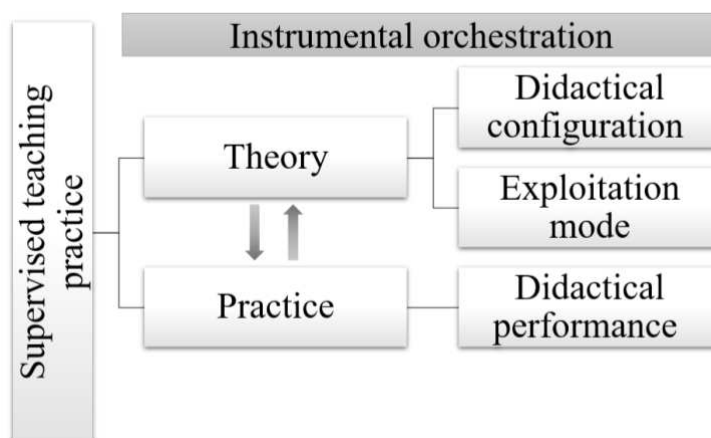
INSTRUMENTAL ORCHESTRATION AND SUPERVISED TEACHING PRACTICE

According to Fiorentini and Lorenzato (2012) although teaching and research are inseparable practices, they are distinct and, therefore, have different objectives. Thus, it is necessary to delineate these differences.

We can say that, as an educator, the teacher's objective is to develop an innovative pedagogical practice in mathematics (exploratory, investigative, problematizing, critical, etc.) that is as effective as possible from the point of view of student education/formation. However, as a researcher, the teacher's objective is to systematize, analyze, and understand how the students' educational process takes place or what the limits and their didactic-pedagogical potential of this innovative practice are. In other words, the research aims to extract lessons, learnings, or knowledge from teaching experiences (FIORENTINI; LORENZATO, 2012, p. 76).

Such an understanding of the role of teaching and research in the teacher's professional practice underpins our proposition of thinking about the model of *instrumental orchestration* and, in particular, of the *online instrumental orchestration* as a theoretical-methodological foundation to guide pedagogical practices developed by pre-service teachers in the classroom in the context of the supervised teaching practice.

Figure 3 – The supervised teaching practice in the light of the IO model



Source: Authors (2022).

Figure 3 depicts the elements of IO in the *theoretical field*: didactical configuration and exploitation mode. In the dimension of *didactical configuration*, the pre-service teacher can ask: What should I address in terms of content? What is the didactic objective of my pedagogical practice? Which artifacts will I use: digital or analog? What is the teaching modality: face-to-face or online? How long is the class? What are the roles of the subjects involved: teacher and students? What are my expectations for implementing IO in the classroom?

These questions lead us to the dimension of the *exploitation mode*, favoring other questions: How can IO contribute to students' development of instrumental genesis? How will teaching planning be conducive to student learning? How will the IO be performed? What are the moments of its exploitations? What do I expect from the discussions, reflections, and actions expressed by the students? What are my expectations about the student reaction to the tasks?

We consider that in the *practical field*, the OIO component *didactical performance*, through which the implementation of what was planned occurs (in terms of didactical configuration and exploitation mode), what the teacher or pre-service teacher predicted appears, as well as unexpected reactions and decisions, given that in the classroom, we have not only the teacher's expectations but also the students' singularities. Thus, the practical part of the IO is essential to understand the potential of the teacher's and/or pre-service teacher's planning and intentional organization and the possible advances in their work. Consequently, in the scenario of the supervised teaching practice, the relationship between the pre-service and the teacher-supervisor assumes an important role in the planning of the orchestrations and the moments of the classes, due to the reflections of both about the professional teaching practice.

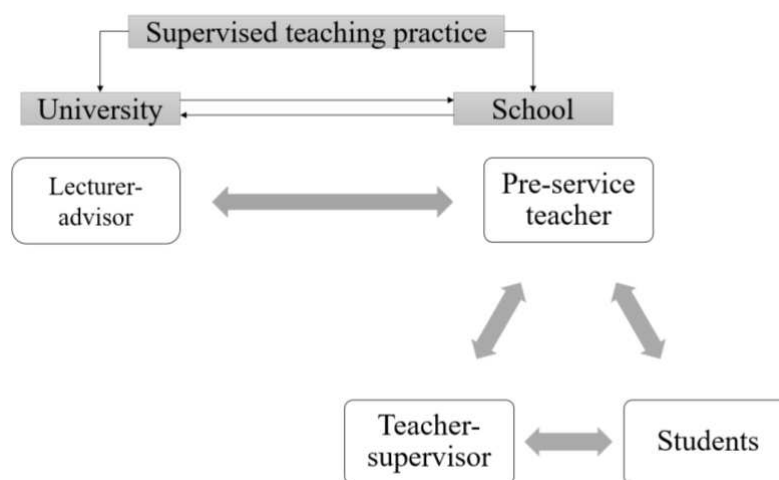
We emphasize that all the planning of the *didactical configuration* and the *exploitation mode* concern the *theoretical field*, as the pre-service teacher needs to mobilize professional knowledge *for* the pedagogical practice. In the *didactical performance*, which refers to the *practical dimension*, the pre-service teacher needs to mobilize professional knowledge *in* the pedagogical practice.

In this sense, it should be noted that we separated the IO components into theoretical and practical dimensions by a methodological convention designed for the supervised teaching practice. This does not imply that there is no practice in the *didactical configuration* and the *exploitation mode*, because, although these two components are previously planned, unforeseen events occur in the classroom given the range of subjectivities of the students. Similarly, the

didactical performance is not devoid of theory, since the orchestration is intentionally planned in the light of the IO model, which guides the pre-service teacher in the didactic decisions.

In this study, we can affirm that the process of planning the orchestrations implemented in mathematics classes, in the context of the supervised teaching practice, started in the relationship between the lecturer-advisor and the pre-service teacher around the study of the theoretical assumptions of the IO and the OIO.

Figure 4 – Supervised teaching practice



Source: Authors (2022).

As we can see in figure 4, the supervised teaching establishes relations between the university (through the collaboration between the lecturer-advisor and the pre-service teacher) and the school (through collective work between the pre-service teacher, the teacher-supervisor, and the students), which requires the practice of planning, executing and reflecting on the class.

In the context of this research, the pre-service teacher elaborated online instrumental orchestrations for the study of equality relations in partnership with the teacher-supervisor of the school. Therefore, the knowledge about the IO model (supervised by the lecturer-advisor) and the observations of online mathematics classes (supervised by the teacher-supervisor) were essential to improve the pre-service teacher's planning.

METHODOLOGY

The type of research adopted in this study was action research. According to Fiorentini and Lorenzato (2012), this kind of investigation consists of articulating investigative practice, reflective practice, and educational practice.

Action research is a special type of participant research, in which the researcher enters the environment to be studied not only to observe and understand it but, above all, to change it in directions that allow for the improvement of practices and greater participants' freedom of action and learning (FIORENTINI; LORENZATO, 2012, p. 112).

Action research is characterized by the in-depth analysis of data phenomena and not by the quantitative generalization of large samples. Consequently, the nature of this investigation is qualitative, as it “occupies a recognized place among the various possibilities of studying the phenomena that involve human beings and their intricate social relationships, established in different environments” (GODOY, 1995, p. 91).

As mentioned above, we aim to discuss the use of the notion of online instrumental orchestration as a theoretical-methodological proposal for the development of classes in the context of the supervised teaching practice. To achieve this aim, we present part of the results of the research undertaken by Almeida (2022) regarding the analysis of the implementation of online instrumental orchestrations to study equality relations in a remote class with students from the 6th grade of elementary school.

The research was developed in a federal public school in the context of the supervised teaching practice of the mathematics teaching degree at the Federal Rural University of Pernambuco (UFRPE), both located in the city of Recife, in the state of Pernambuco. Thirteen students from the 6th grade of elementary school, a pre-service teacher and the teacher-supervisor participated in this investigation.

The class we discuss in this article occurred in the remote teaching scenario driven by the demands brought about by the Covid-19 pandemic. It was planned to be in the format of a composition of three immediate sequenced OIOs (figure 5) to be performed one after the other, that is, with a sequential time, as indicated by Lucena (2018).

Figure 5 – Composition of immediate sequenced OIOs



Source: Almeida (2022).

For the implementation of this composition of the immediate sequenced OIOs (figure 5), we planned a synchronous one-hour class through Google Meet, to be held in real time, requiring participants to have access to electronic devices (laptop, computer, cell phone, or

tablet) and the internet. In this article, due to necessary limitations, we present only the *OIO “technical-demo”* and the *OIO “discussion between stakeholders”*, concerning the use of a particular artifact in mathematics class: the *Equality Explorer: Basics* (EEB) available on the PhET platform (UNIVERSITY OF COLORADO BOULDER, 2022).

In the data analysis procedures, we used the pre-service teacher's logbook, where the registers for data construction records were. In this diary, the pre-service teacher described before, during, and after the class, the main moments of the OIO, primarily those that allowed expanding the discussion on the equality relations in the teaching of school algebra, and specific factors of the remote teaching modality (interactions via chat, microphone, etc.). We emphasize aspects such as didactical performances by identifying *ad hoc reactions* and *ad hoc decisions* that resulted from the unforeseen events that occurred against what we had planned.

Furthermore, we restated the screenshots of the simulations proposed by the pre-service teacher and the students with the *Equality Explorer: Basics*, and the screenshots of some moments of the class. Such captures were taken by the pre-service teacher during the class and later edited to preserve the anonymity of the participants. Therefore, in the class captures presented in the results, we have the pre-service teacher (identified with the “orange face”); the students (identified with the “little blue face”) and the teacher (identified with the “little green face”). In the case of participants with the camera on, we have a “white background” behind the “little face”. Those captures were fundamental to illustrate the research results. We also pointed out that, for the participation reports of some students who interacted in the class, we used codenames to preserve their identity.

RESULTS AND DISCUSSION

OIO “technical-demo”

In the *OIO “technical-demo”*, the pre-service teacher introduced the students to the EEB tools and their functionalities. So, we have the following *class of situation*: the use of the *Equality Explorer: Basics*. And the *learning objectives* were: 1st) Knowing the ostensive aspects of the EEB; 2nd) Understanding the tools and functions of the EEB; 3rd) Understanding how to perform interactive simulations with the EEB. Regarding the didactical configuration, we highlight that the main artifacts used in this OIO were: *Equality Explorer: Basics* through the PhET platform and videoconferencing through Google Meet. It lasted eight minutes.

The *OIO “technical-demo”* was organized in three moments: 1st) a general description of the EEB; 2nd) a presentation of the technical aspects of the basics mode; 3rd) a presentation of the technical aspects of the lab mode. In chart 1, we describe the relations between: the artifacts, the roles of the subjects involved, the main actions, and the duration. In addition, we list the moments of these relations in the *OIO “technical-demo”*.

Chart 1 – OIO didactic configuration “technical-demo” implemented

Artifacts	Roles of the pre-service teacher, the teacher-supervisor, and the students	Actions	Time	Moments
Equality Explorer: Basics through the PhET platform + Videoconferencing through Google Meet	<p>Pre-service teacher: provide the EEB link through the Google Meet chat and present general descriptions of the digital resource.</p> <p>Students: answer the question proposed by the pre-service teacher.</p> <p>Teacher-supervisor: observe the progress of the class.</p>	The pre-service teacher provides the EEB link, asks whether the students know the digital artifact and presents them with general information on the EEB.	2 min.	1 st moment
Equality Explorer: Basics through the PhET platform (basics mode) + Videoconferencing through Google Meet	<p>Pre-service teacher: Share the EEB screen with students through Google Meet, and present its tools and functions in basics mode.</p> <p>Students: Answer the questions proposed by the pre-service teacher and ask questions about the tools and features of the artifact.</p> <p>Teacher-supervisor: Intervene at necessary times, propose questions and help with class performance.</p>	The pre-service teacher presents the EEB tools in basics mode, and their functions. If the students don't understand something, they ask the pre-service teacher.	2 min.	2 nd moment
Equality Explorer: Basics through the PhET platform + Videoconferencing through Google Meet	<p>Pre-service teacher: Share the EEB screen with students through Google Meet, and present its tools and functions in laboratory mode.</p> <p>Students: answer the questions proposed by the pre-service teacher and ask questions about the tools and features of the resource.</p> <p>Teacher-supervisor: Intervene at necessary times, propose questions and help with class performance.</p>	The pre-service teacher presents the EEB tools in lab mode and their functions. If the students don't understand something, they ask the pre-service teacher.	4 min.	3 rd moment

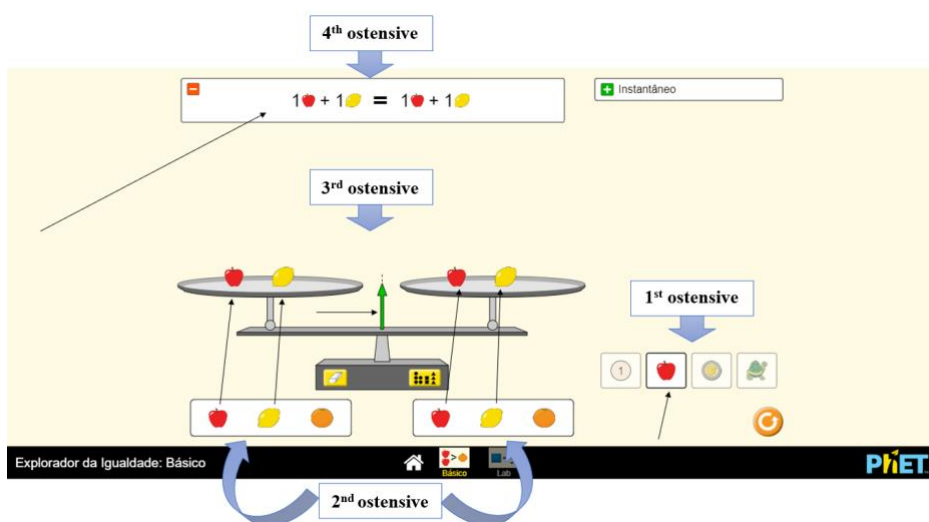
Source: Almeida (2022).

At first, the pre-service teacher introduced the *OIO “technical-demo”*, explaining to the students the function of the EEB: to work on the idea of balance and imbalance in analogy to the notions of equality and inequality, respectively, through situations based on interactive simulation. Then, the pre-service teacher provided the EEB link available on the PhET platform, through Google Meet chat, and told them to access and explore the digital resource concomitantly with the explanation in class.

In the second moment, the pre-service teacher sought to present the main tools of the digital artifact, the ostensive aspects: O_1 – *Types of objects*; O_2 – *Objects to be inserted into the scale*; O_3 – *Double-pan scale*; and O_4 – *Equal sign between two sentences*.

In the technical demonstration of EEB in basics mode, the object type (O_1), chosen by the pre-service teacher to be included in the scale, was “fruits”. For the simulation, one apple and one lemon (O_2) we inserted on both pans of the scale (O_3). In this example, we have an equality situation for the equality relation, since both members have equal objects. With this understanding, the pre-service teacher points with the mouse arrow to the green arrow of the scale to signal that it is balanced. Then he has the students verify that when the scales (O_3) are in balance, we can represent this situation in a mathematical sentence using the “=” sign, pointing with the mouse arrow to the ostensive 4. We illustrate this situation in figure 6:

Figure 6 – EEB technical demonstration in basics mode



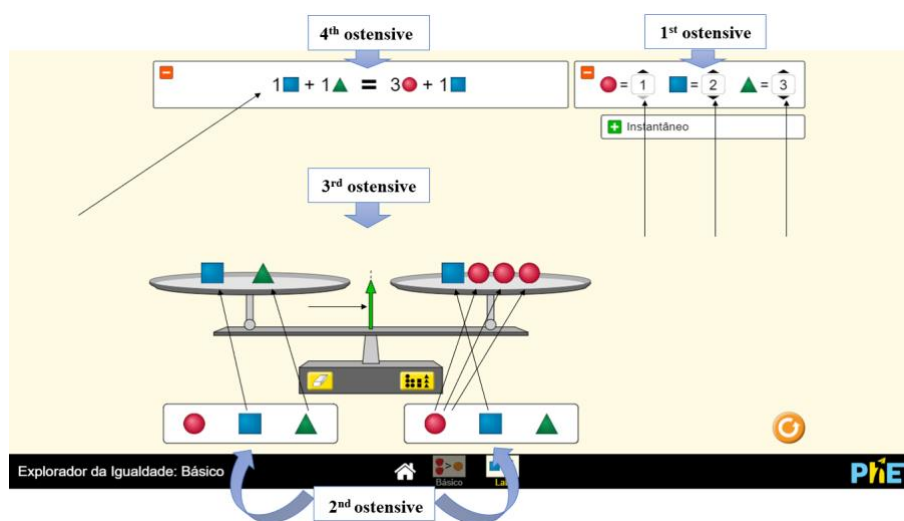
Source: Research collection (2022).

In the technical demonstration of the EEB in the lab mode, the pre-service teacher initially highlighted that the objects (O_1) available range from a value of 1 to 20. He also introduced the capture tool since, later on, he would need it to save the discussed situations. For

the simulation, the pre-service teacher inserted the cube and the tetrahedron (O_2), on the left side of the scale, with the respective values: 2 and 3. In addition, he asked students to look at ostensive 1 to understand how equilibrium occurs on the scale. Afterwards, the pre-service teacher asked the students what to put on the right side of the scale. A student answered on the microphone: a cube and a tetrahedron. In this scenario, it is possible to notice that the student gave a trivial answer and proposed the reflexive property of equality. In other words, he proposed a situation of equality.

After that, a male and female student answered the following: a cube and a sphere. The pre-service teacher asked whether that was all it was, pointing out that the scale was unbalanced. Students told him to add two more spheres (O_2) on the right side. And thus, a balance was obtained between the two sides of the scale (O_3). From this, the pre-service teacher explained that there was a situation of equivalence for the presented equality relation. We present the following illustration of this situation:

Figure 7 – EEB technical demonstration in lab mode



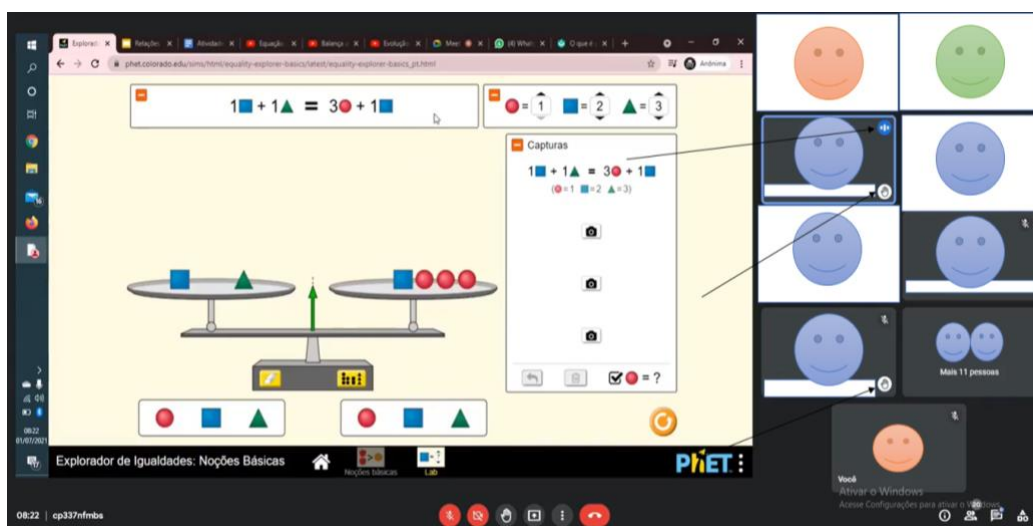
Source: Research collection (2022).

Finally, as in the technical demonstration in the basic mode, the pre-service teacher led students to realize that when the scales (O_3) are balanced, it is possible to represent this situation by means of a mathematical sentence involving the “=” sign, pointing with the mouse arrow to the ostensive 4.

Then, the teacher-supervisor informed the pre-service teacher that the student William had raised his hand (referring to the Google Meet), as seen in figure 8. This occurrence clearly illustrates the teacher's importance in helping the class, because his intervention was

fundamental for the pre-service teacher to know what was happening in the videoconference since he was focused on the technical demonstration of the EEB.

Figure 8 – Example of the *OIO* “technical-demo” at the 3rd moment



Source: Research collection (2022).

When the pre-service teacher asked William to comment, he turned on the microphone and informed that he had the artifact open on his computer, but he was unable to access the page. The pre-service teacher asked on which page he was, and the student said he could not access laboratory mode. Thus, the pre-service teacher provided the link to the page, again, on the Google Meet chat. And this time, William managed to access it. At the end of the *OIO* “technical-demo”, student Anna, who also had her hand raised (see figure 8), opened the microphone and said that she could find an equality relation. The situation presented by the student was the starting point for the realization of the *OIO* “discussion between stakeholders”, which we detail below.

OIO “discussion between stakeholders”

In the implementation of the *OIO* “discussion between stakeholders”, the students proposed examples of equality relations, based on the interactive simulations with the EEB, through which the pre-service teacher sought to deepen the discussions given the elements of mathematical organizations around the use of the digital artifact for the study of the theme. Finally, the pre-service teacher proposed a problem, so that the students could use the digital artifact as part of the resolution techniques.

Therefore, the *class of situation* on stage was: the elaboration and resolution of problem situations about equality relations using the *Equality Explorer: Basics*. And, the *learning objectives* were: 1st) going through the process of instrumental genesis with the EEB; 2nd) understanding the principles of equivalence; 3rd) understanding the properties of equality; 4th) calculating unknown terms in the equality relations.

In chart 2, we establish relations between the following aspects of an OIO: the *artifacts*, the *roles of the subjects involved*, the *main actions*, and the *duration*. Furthermore, we list the moments of the OIO “*discussion between stakeholders*” that help in its description for a better understanding of these relations.

Chart 2 – OIO didactical configuration “discussion between stakeholders” implemented

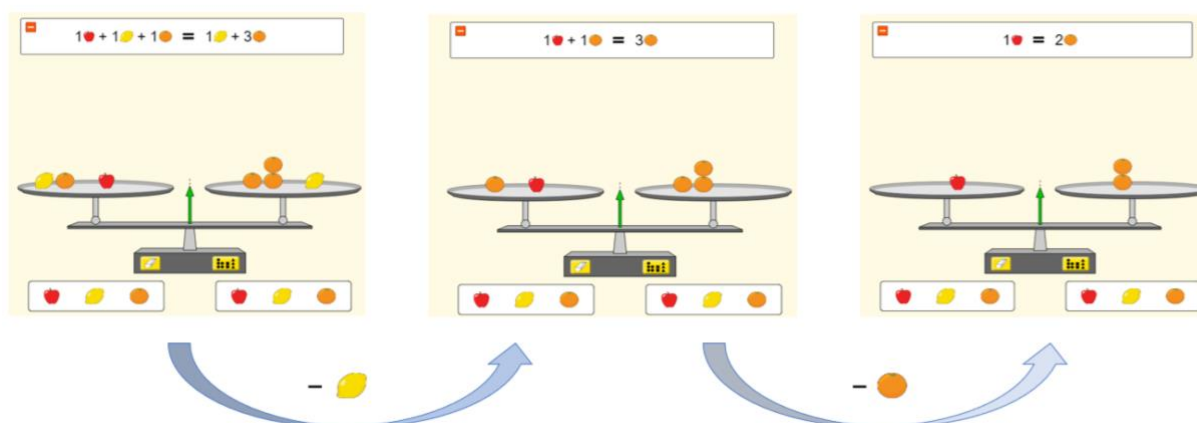
Artifacts	Roles of the pre-service teacher, the teacher-supervisor, and the students	Actions	Time	Moments
Equality Explorer: Basics through the PhET platform + Videoconferencing through Google Meet	<p>Students: find and propose examples of equality relations in the basics mode to discuss with the class.</p> <p>Pre-service teacher: to mediate the discussions and substantiate the students' arguments to explain the content.</p> <p>Teacher-supervisor: intervene at necessary times, propose questions and help students and pre-service teacher.</p>	Students propose examples with the EEB. To do so, they describe step by step what must be done, for the pre-service teacher to carry out the commands and share the screen with the situation for everyone.	8 min.	1 st moment
Equality Explorer: Basics through the PhET platform + Videoconferencing through Google Meet	<p>Pre-service teacher: resume the example of the technical demonstration of the lab mode of the EEB, to discuss with the class.</p> <p>Students: answer the questions proposed by the pre-service teacher and clarify doubts about the situation under discussion.</p> <p>Teacher-supervisor: intervene at necessary times, propose questions and help students and pre-service teacher.</p>	The pre-service teacher resumes an example to deepen the discussions. Throughout this moment, he questions the students to generate discussions. Students interact by answering the questions.	8 min.	2 nd moment
Presentation through Google Slides + Equality Explorer: Basics through the PhET platform (basics mode) +	<p>Pre-service teacher: propose a problem situation to be answered by the students with the help of the EEB.</p> <p>Students: interpret the problem and simulate on the EEB to find the solution. Answer the questions proposed by the pre-service teacher and clarify doubts about the situation under discussion.</p> <p>Teacher-supervisor: intervene at</p>	The pre-service teacher presents a problem situation to the students for them to solve.	20 min.	3 rd moment

Videoconferencing through Google Meet	necessary times, propose questions and help students and the pre-service teacher.			
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Source: Almeida (2022).

At first, student Anna told the pre-service teacher that she found the following: 1 apple + 1 lemon + 1 orange = 3 oranges + 1 lemon. The pre-service teacher thought that the students could simplify the example presented by Anna, asking how much an apple was equivalent to. Anna said there were two. George said there were three. Given this, the pre-service teacher asks George and Anna about what they need to do to verify the answer. Anna said to remove a lemon from one side and the other. Then repeat the same process, taking out an orange on both sides of the scale. In this way, they reached the answer that Anna had given. The commands Anna gave the pre-service teacher indicate the student's understanding of the equality relation in a situation of equivalence. This situation is shown in figure 9:

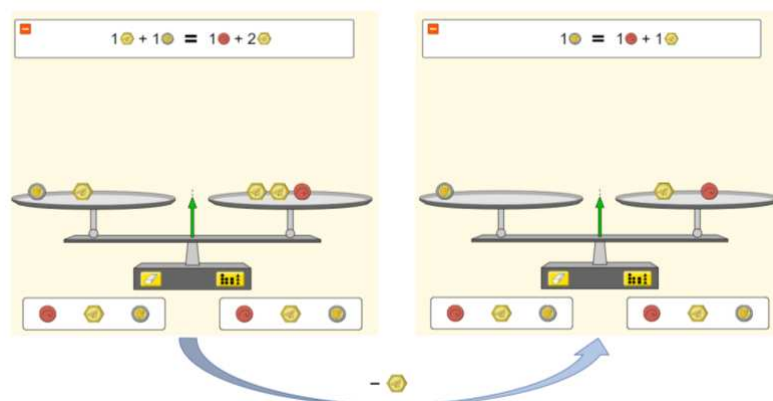
Figure 9 – Example of an equality relation presented by Anna



Source: Research collection (2022).

Another example presented was by the student Oliver. He said that he had also found an equality relation with coins. The pre-service teacher asked how he had found it and he said that he had placed the coins with a “gray border” and with a “little airplane” on top of one of the plates and, on the other, two coins with a “little airplane” and a “red one”. Again, the pre-service teacher pointed out that students could simplify the example presented by Oliver. And, immediately, Anna and Oliver replied that it was only necessary to remove the coin with the “little airplane” on both sides of the scale. Thus, we have again that when Anna and Oliver tell the step-by-step of what the pre-service teacher should do, there are indications of the students' understanding of the equality relation in a situation of equivalence.

Figure 10 – Example of an equality relation presented by Oliver



Source: Research collection (2022).

Using the example of figure 10, the pre-service teacher reinforced that in mathematical equality and in the double-pan scale – when you remove a coin from one side, you also have to remove the same from the other –, you need to operate on both sides, so that the equality relation remains; obtaining another one equivalent to the previous one.

An unexpected episode by the pre-service teacher during this OIO was that, in the middle of the class, a student came in commenting on the chat that she took a while to log in because she had trouble turning on the computer. She asked for the page and the teacher-supervisor replied that the activity was not in the book. In this situation, it is possible to perceive that this event (the student's *ad hoc* reaction) the pre-service teacher had not foreseen required that the teacher-supervisor made a didactic decision since the pre-service teacher was mediating the discussion with the other students.

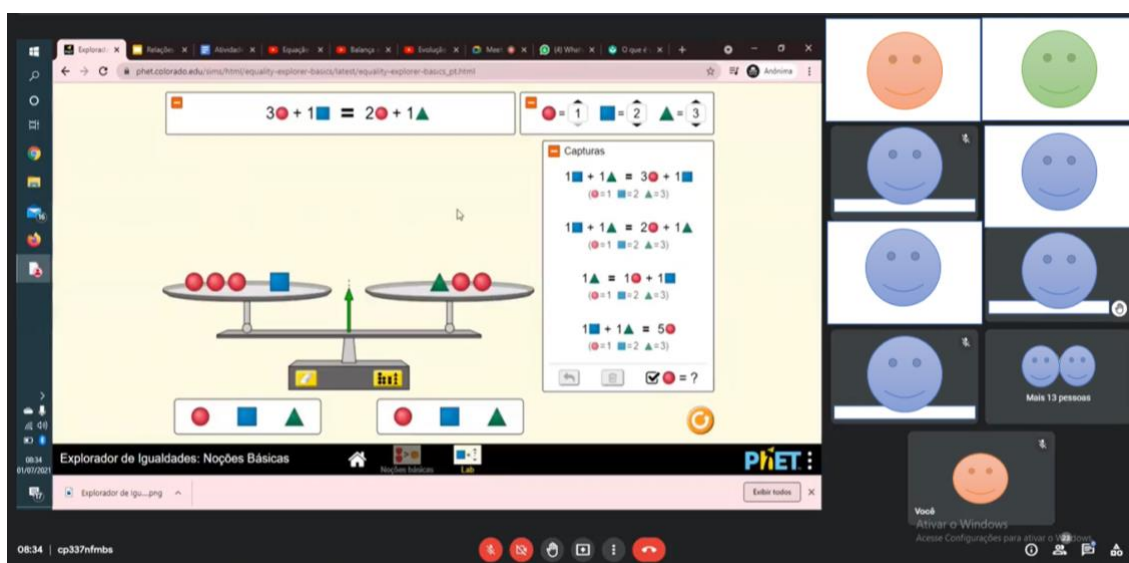
In summary, in this first moment of the OIO “*discussion between stakeholders*”, we noticed that some students carried out simulations individually on their electronic devices and proposed to the pre-service teacher, through commands on the microphone, that he conduct the simulations and present to the class by sharing the screen on the Google Meet. From the examples the students proposed, the pre-service teacher deepened the discussions, bringing mathematical arguments for the use of techniques in solving the tasks.

In the second moment, the pre-service teacher returned to the example of the technical demonstration of the lab mode of the EEB, to deepen the discussions on the equality relations. At that moment, the pre-service teacher asked the students to present equality relations equivalent to the initially proposed equality relation. Thus, the examples led him to emphasize

the transitive property, so that students could start from the premises (two initial equality relations) to conclude (another equivalent equality relation).

Resuming the example presented in the *OIO* “technical-demo” in lab mode, the pre-service teacher sought to work on other relations of equivalent equality, with objects with the same values. George and Anna proposed the second equality relation of the captures window. Oliver proposed the third equality relation. They all instructed the pre-service teacher to insert the objects on the scale.

Figure 11 – Example involving the transitive property of equality



Source: Research collection (2022).

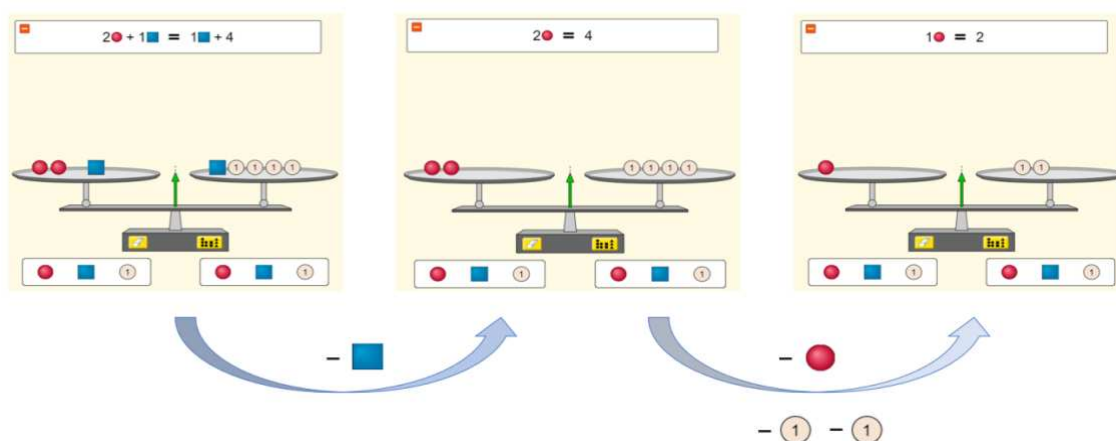
In the meantime, Aurora tried to present another example in basics mode. However, the pre-service teacher asked to conclude this situation and then return to the example given by Aurora. This student's *ad hoc* reaction allowed another rearrangement of this OIO, because it required the pre-service to resume the students' moment of proposing examples, foreseen as the first to be carried out, instead of moving on to the third moment.

Continuing the example in figure 11, the pre-service teacher asked the students what conclusion they could draw from the first and second equalities in the captures window. George said it was to put five balls on the right side of the scale. Again, the pre-service teacher rephrased the question, as George presented another equality relation. However, he had not yet reached the transitive property. Anna tried to answer, but could not. The pre-service teacher exemplified by inserting the objects on the scale and concluded.

Realizing the students' difficulty in answering the problem, the pre-service teacher takes up Anna's example of working on the transitive property. He explained on the mic that, as proposed by Anna at the beginning of the class: $7 + 5 = 12$ and $4 \times 3 = 12$. So, it is possible to conclude that $7 + 5 = 4 \times 3$. In this context, a student's connection was not good and he asked the pre-service teacher to repeat it. Immediately, the pre-service teacher repeated it and the student said he understood the example.

Resuming Aurora's proposal, the pre-service teacher asked her to explain what she had done and she proposed the following situation: 2 red balls + 1 blue cube = 1 blue cube + 4 (figure 12). Anna said she understood that 1 red ball equals 2 beige balls. The pre-service teacher asked how she could understand this and she said that she removed 1 cube from each side of the scale. Then, she stated that it was necessary to divide the 4 beiges by 2 and the 2 reds by 2, i.e., half was removed on both sides.

Figure 12 – Example of an equality relation presented by Aurora



Source: Research collection (2022).

In the third and last moment of the *OIO* “discussion between stakeholders”, the pre-service teacher presented a problem situation to the class on slides, for the students to interpret the problem and use the EEB in basics mode to answer it. Due to some difficulties the students had in interpreting the problem, the teacher intervened in a specific situation and worked on it.

In general, from the commands given by some students to the pre-service teacher in the interactive simulations with the *Equality Explorer: Basics* we identified evidence of their *instrumental genesis*. This is because when students use the digital artifact to simulate in search of examples of equality, they go through the *instrumentation process*, in which they explore

tools and functions of the artifacts guided by their prior knowledge. Furthermore, these students go through the *instrumentalization process* when they understand the technical aspects of the artifact and begin to use it to propose and solve problem situations.

Roughly speaking, other important events occurred in the implementation of the orchestrations; for example, the episode in which the teacher-supervisor explicitly intervened in a situation in which a student had learning difficulties. Those events are found in more detail in the work by Almeida (2022).

FINAL CONSIDERATIONS

In this article, we aimed to discuss the use of the notion of online instrumental orchestration as a theoretical-methodological proposal for the development of classes in the context of a supervised teaching practice., driven by the demands brought about by the Covid-19 pandemic, in the remote teaching scenario. Thus, we highlight the importance of systematically planning the arrangement of the teaching environment for online classes, taking into account the use of digital artifacts for mathematics learning. In this regard, we mentioned that the previous use of the *Equality Explorer: Basics* by a pre-service teacher made it possible to elaborate the *didactical configurations* (charts 1 and 2) and the *exploitation modes*, as well as to do *ad hoc decisions* considering the mathematical situations presented by the students in didactical performances during the class.

We emphasize that, despite presenting excerpts from the results of a broader research, we intend to promote reflections on the roles of the pre-service teacher, the teacher-supervisor, and the students, in the school environment, through the *OIO “technical-demo”* and the *OIO “discussion between stakeholders”*, as well as the relationship between the pre-service teacher and the lecturer-advisor in the planning and reflection on the implementation of orchestrations.

We consider that the discussions undertaken in this article provide evidence of new perspectives on becoming a teacher who investigates their own educational practice, reinforcing the relevance of experiences in the supervised teaching practice for the professional development of teachers by assuming the notion of online instrumental orchestration as a theoretical-methodological proposal in initial education. Moreover, we believe that the open perspectives on synchronous and asynchronous classes, widespread in remote teaching, are plausible for further research given the necessary reflection on planning for the use of digital artifacts, especially in online classes.

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