CHISHOLM'S MODAL PARADOX: TWO APPROACHES EXAMINED

O paradoxo modal de Chisholm: examinando duas abordagens

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ABSTRACT

This paper is organized in the following way: the first section puts forward the version of Chisholm's Modal Paradox that is going to be taken into consideration. The presentation of the paradox is followed by two sections where two *invalidating strategies* (as I call them) are presented; the second section presents Forbes' strategy for dealing with the paradox. As part of the second section, it is presented Forbes' many-valued semantics for modal logic. The third section is a presentation of Salmon's strategy to solve the paradox. The fourth and last section is reserved for critical comments concerning both approaches. Those comments are to be understood as motivations against each strategy not as definitive refutation of either of them. I finish the paper concluding that Salmon's strategy seems to be the most conservative invalidating strategy.

Keywords: Chisholm's paradox. Modality. Many-valued logic. Transworld identity.

RESUMO

Este artigo é organizado da seguinte maneira: a primeira seção é a apresentação da versão do Paradoxo de Chisholm que será discutida ao longo do artigo. A apresentação do paradoxo é seguida por duas partes onde são apresentadas duas versões do que chamo *estratégia de invalidação*; na segunda seção é apresentada a estratégia do Forbes, incluindo a lógica de multi-valores apresentada por ele. E na terceira seção é apresentada a estratégia do Salmon para solucionar o paradoxo. A quarta e última seção apresenta duas objeções a cada uma das soluções. Essas objeções não devem ser encaradas como argumentos definitivos, mas devem ser adicionadas ao cálculo para a escolha da estratégia de invalidação que parece oferecer a melhor solução ao paradoxo ao menor custo teórico. O artigo termina com uma conclusão sugerindo que a solução de Salmon parece ser a estratégia de invalidação mais conservadora.

Palavras-chave: Paradoxo de Chisholm. Modalidade. Lógica de multi-valores. Identidade transmundial.

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Introduction

We intuit that a particular bicycle which, in fact, came into existence made up of parts $P_1, P_2, P_3, \dots, P_n$ could not have come into existence made up of totally different parts. On the other hand, this bicycle *could* have come into existence with *one* of its parts different from the one it actually had. Consider a possible world in which the bicycle now before us came into existence with a different spoke. Surely *that* bicycle could have come into existence with, say, a different handle-grip than one of the ones it did have at its origin, there, in that possible world. That is to say, we now see a second possible world in which the bicycle I indicated in the first possible world came into existence with a different handle-grip. Proceeding in this fashion, we seem to work our way towards a world in which our bicycle came into existence made entirely of parts other than $P_1, P_2, P_3 \dots P_n$.

‘Chisholm’s Paradox’ may be understood as denoting a family or a class of paradoxes regarding modality; possibility and necessity or, more precisely, paradoxes related to *de re* modality, i.e., *de re* possibility and *de re* necessity. Chisholm himself has presented some different versions of the paradoxes in different places (1967) (1973) and many other philosophers have advanced their own versions Chandler (1976), Forbes (1986) (1984), Salmon (1986) (1989), Fara (2012), Leslie (2011) and Williamson (1990) just to name a few. Similarly, many philosophers have offered their own solution to the paradox. So, as expected, there are a couple of different strategies out there to deal with the paradox. In this paper, we are going to look into two of these different strategies: on the one hand, the *counterpart relation strategy*, which is represented here by Forbes’ approach, which combines a sort of *many-valued semantics* for modal logic and a *counterpart relation* between objects in different possible worlds. On the other hand, the *transworld identity strategy*, which is represented here by Salmon’s solution, in which, basically, combines the standard transworld identity relation of the *Kripke semantics* for modal logic with a *non-transitive relation* between possible worlds. Both *the counterpart relation strategy* and *the transworld identity strategy* (as they are understood here) are just representatives of their kinds. There are, once again, many alternative implementations of each of them available in the literature. The same remark applies to the versions of the paradox: the paradox presented here is just one of many of its kind.

One interesting common feature of Forbes' and Salmon's strategies is their focus on the *formal logic of the paradox*, i.e., the logical principles that allow one to move from premises to conclusion of the paradoxical argument. Both accept the premises of the argument as true and then try to show that the underlying logic is *fallacious* or, at the very least, inadequate to express Chisholm's Paradox. Exploring these 'invalidating strategies' (as I call them) is the main aim of this paper. An invalidating strategy, instead of disputing some premise of an argument, argues against the formal system which supposedly *validates* it; Forbes' targets are *bivalence* and *transworld identity* and Salmon's target is *transitivity* of accessibility relation between possible worlds.

This paper is structured as follows: in the first section, I put forward the version of Chisholm's Paradox that is going to be taken into consideration. The formulation considered here is inspired by Chisholm (1967) (1973), although it is not Chisholm's own formulation. There are three reasons for doing that: i) neither Forbes nor Salmon are dealing with Chisholm's own formulation, each one offers his own version, ii) the formulation chosen here is stronger (meaning that providing a solution to the current version implies that a solution to Chisholm's original formulation does indeed exist) and iii) the formulation in hand is accepted by both Salmon and Forbes. The second section presents Forbes' strategy of dealing with the paradox. In order to present Forbes' strategy, I will first develop the set of formal tools used by Forbes to create his alternative semantics to modal logic (and first-order logic) that is supposed to solve a broader set of paradoxes (including the more familiar Sorites Paradox). The third section analyses Salmon's strategy to solve the paradox. As we will later see, Salmon's solution is indeed a lot simpler than Forbes', which does not necessarily mean that Salmon's approach is better, even though one might take simplicity as one important virtue. The fourth and last section of the paper puts forward some critical comments concerning both approaches and argues that Salmon's strategy is the most conservative.

The Paradox

In this section, I am going to present the formulation of the modal paradox that we are interested in. The main aim of this section is not to critically discuss possible solutions to the paradox, but I will try to provide the reader with the necessary background to be able to follow the solutions that will be advanced. As I have said, there are many different versions of the paradox. The version which will be presented here has deliberately been formulated to simplify the presentation of both Forbes' and Salmon's strategies. In spite of being inspired by Chisholm's work (1967) (1973), the version of the paradox considered here involves essentialist principles that were not employed by Chisholm himself. Those principles are accepted by both Forbes and Salmon and they will make the whole paper significantly simpler, focusing our attention on the differences between the alternative accounts. So, before moving on to the paradox itself, let us have a look at the underlying essentialist framework.

Something about the essentialist framework

The paradox that we are going to analyse is explicitly related to essentialism regarding the original material composition of an artefact. More generally, it is associated with an essentialist thesis about the origin of an artefact (a table, for instance), which has been commonly endorsed by many essentialist philosophers since its famous formulation by Kripke (1980, 114):

Could *this table* have been made from a completely *different* block of wood? [...] We can imagine making a table out of another block of wood, identical in appearance with this one, [...] it seems to me that this is *not* to imagine *this* table as made of [different] wood, but rather it is to imagine another table, *resembling* this one in all external details, made of another block of wood.

Many philosophers who accept the essentialist claim frequently acknowledge the relevance of the paradox, even though it is not clear whether Kripke himself is convinced of its relevance.

My main aim here is just try to throw some light on the conceptual framework connected to the modal paradox and modality itself.

The modal paradox arises from the following two essentialist intuitions:

[E₁] The original matter of an artefact is essential to it.

[T] Slight changes in the original matter of an artefact are tolerable.

Essentialist intuitions E₁ and T are supposed to be equivalent to the following two modal principles:

[E₂] A complete or substantive change in the original matter of an artefact is not possible.

[M_T] Slight changes in the original matter of an artefact are possible.

E₁ and E₂ seem to be equivalent. E₁ says that for any object *O* originally made of *m* and any possible world *w*, *O* exists in *w* only if *O* is made of *m* in *w*. Even though it may suggest that all matter is essential, it is not entailed by E₁ (in a weak reading). E₂ says something quite similar; there is no possible world where *O* was originally made of *n*, matter completely different of *m*. Adopting the weaker reading for E₁, it seems to be equivalent to E₂. So, you can feel free to choose which seems textually more plausible. M_T (modal tolerance) just says that it is possible for *O* to be originally made of a slightly different matter. So, there is a possible world where *O* was originally made of a slightly different matter. Although T (tolerance) does not make any explicit mention to modality, it may (in this context) be understood as saying that it is possible for *O* to be made of slightly different matter; exactly the same that is meant by M_T. Thus, the two pairs of principles will be interchangeably used throughout this paper, since *prima facie* they do not differ substantially in meaning. In what follows, E is going to be used to refer to both E₁ and E₂ and T analogously is going to be a label to T and M_T.

Deriving a paradox

In this section, I will provide a formal derivation of an explicit paradox from the modal principles above following explicit steps allowed by modal logic. Just by doing that, it will become clear how strong the (alethic) modal logic must be to give rise to the paradox.

There are several different ways to derive an explicit paradox from the modal principles E and T. In what follows, we will be faced with one of them. The paradox that I am going to present is composed of two stages that drive us from the set up (basically, the modal principles just mentioned) to a contradiction through allegedly uncontroversial steps. The first one, the tolerance stage, reflects the semantic idea of the relevant object slowly changing its material origin through a line of possible worlds in such a way that it is allowed by T. The second one, the S4 stage, is the stage where the object of one far away world is shown to be possible by the distinctive modal principle of **S4**, which says that if something is possibly possible, then it is possible and so on. That is exactly the English expression of the role performed in modal logic by the converse of the characteristic axiom schema of **S4** $\Diamond\Diamond\Phi \rightarrow \Diamond\Phi$, which states that any formula (including, obviously, formulae with prior modal operators) that is possibly possible is possible.

In the following derivation, ‘M’ is a dyadic predicate meaning ‘originally composed of’ and ‘M__’ is to be read as ‘__ is originally composed of __’. The constant *a* is used to refer to a specific object (a table, for instance) and *m*, similarly, is used as a name for a specific hunk of matter. Thus, ‘Mam’ should be read as ‘*a* is composed of *m*’. In what follows, underwriting is used to make available an infinitely large stock of letters to refer to specific hunks of matter and overwriting is indicating the iteration of modal operators. The underwriting is indicating a list of slight changes in the material composition of the objects. So, the matter m_n is to be understood as being as similar as possible of the matter m_{n-1} and so on. And, finally, the logical connectors are the usual ones of modal logic.

- 1) Mam₀
- 2) $\neg \Diamond \text{Mam}_n$ An instance of [E] Set up
- 3) $\Box (\text{Mam}_0 \rightarrow \Diamond \text{Mam}_1)$ An instance of [T]
- 4) $\Box\Box (\Diamond \text{Mam}_1 \rightarrow \Diamond\Diamond \text{Mam}_2)$
- 5) $\Box^3 (\Diamond^2 \text{Mam}_2 \rightarrow \Diamond^3 \text{Mam}_3)$
- 6) $\Box^4 (\Diamond^3 \text{Mam}_3 \rightarrow \Diamond^4 \text{Mam}_4)$ Tolerance stage
- n) $\Box^n (\Diamond^{n-1} \text{Mam}_{n-1} \rightarrow \Diamond^n \text{Mam}_n)$

$$\begin{aligned}
 & n+S4_1) \Box^{n-1} (\Diamond^n \text{Mam}_n \rightarrow \Diamond^{n-1} \text{Mam}_n) \\
 & n+S4_2) \Box^{n-2} (\Diamond^{n-1} \text{Mam}_n \rightarrow \Diamond^{n-2} \text{Mam}_n) \\
 & n+S4_3) \Box (\Diamond \text{Mam}_n \rightarrow \Diamond \text{Mam}_n) \quad \text{S4 stage} \\
 & n+S4_n) \Diamond \text{Mam}_n
 \end{aligned}$$

$$n+S4_{n+1}) \Diamond \text{Mam}_n \wedge \neg \Diamond \text{Mam}_n \quad \text{Contradiction}$$

As has been said, this derivation of the paradox is constructed in just two stages to lead us from the set up (which is basically a formal version of T and E) to the contradiction that says it is both possible and impossible for a specific object a to be made of a hunk of matter m_n (matter of the step n). In spite of not being T itself, step 3 is an instance of one more general modal principle, which presents the following logical form $\Box^n (\Diamond^{n-1} \text{Mam}_n \rightarrow \Diamond^n \text{Mam}_n)$ and we may call it ‘modal principle schema’ (MPS). In the first stage, tolerance stage, MPS is applied n times to reach in the step n the formula $\Box^n (\Diamond^{n-1} \text{Mam}_n \rightarrow \Diamond^n \text{Mam}_n)$. In other words, in the tolerance stage, the relevant object has its material composition slowly changed to reach a matter from which it could not have been made, i.e., it is an impossible way for the relevant object to be. The next stage, S4 stage, aims to show that the supposedly impossible fact reached in step n is not, however, impossible. And it follows quite straightforwardly from the application of S4 modal logic because, although it really is impossible for the object a to be made of m_n , it is possibly possible for a to be made of m_n . Why is possibly possible for a to be made of m_n ? Because there is an intermediate possible way for a to be (m_{n-1}) according to which, if it had been made in such a way, it would be possible for a to be made of m_n . Thus, if it is possibly possible for a to be made of m_n , then, applying the converse of S4 modal logic $\Diamond \Diamond \Phi \rightarrow \Diamond \Phi$ n times, it is possible for a to be made of m_n , which is exactly what is not allowed by the modal principle E. Therefore, we reach in step $n+S4_{n+1}$ an explicit contradiction, so the paradox is demonstrated.

Forbes’ Solution to Chisholm’s Paradox

What follows provides an accessible explanation to Graeme Forbes’ strategy to deal with a version of ‘Chisholm’s Paradox’. As I have men-

tioned, Forbes' strategy is a form of what I have been naming 'invalidating strategy'. As such, he focus on the logical form of the paradox, i.e., the logical principles that allow the move from premises to conclusion of the paradoxical argument. Forbes accepts the premises of the argument as true and then tries to show that the underlying logic is somehow *fallacious* or, at the very least, inadequate to represent Chisholm's Paradox. Forbes' strategy may be summarized in three distinct stages: first, he argues that Chisholm's Paradox is better understood as a special instance of the more familiar Sorites Paradox; a paradox related to the *vagueness phenomenon*. Second, Forbes looks more closely at the underlying logic behind the paradox and develops a solution based on a many-valued first-order logic. Third, Forbes applies exactly the same solution given to the Sorites Paradox for Chisholm's Paradox. In order to do that, the semantics built upon transworld identity – commonly used in standard semantics for first-order modal logic – is replaced by a semantics built upon counterparthood. We will soon return to this point.

Argument: the first stage – Chisholm's Paradox as a Sorites Paradox

In this section, I present some of Forbes' ideas about the logic underlying Chisholm's Paradox, which seems to be closer to the more familiar Sorites Paradox than usually recognized. Forbes' strategy makes explicit the underlying logical form of the ordinary Sorites Paradox and then argues that the standard way of presenting it has an S5-equivalent presentation that, despite running in a modal context, is formally almost the same as the ordinary Sorites Paradox. And finally, he argues that his alternative S5-equivalent formulation should be taken as the more accurate formulation of Chisholm's Paradox. Hence, any successful solution to the ordinary Sorites Paradox could be *mutatis mutandis* successfully applied to Chisholm's Paradox. One may argue that Forbes' alternative formulation is a misleading approach to Chisholm's Paradox, since it may lead us to misdiagnose the problem, associating it with vagueness phenomenon, which might not be the only source of problems. We will return to this point later. For now, let us look into the stages of the argument. As I said above, the first stage of Forbes' strategy

shows that Chisholm's Paradox is an instance of the Sorites Paradox. So, let us first analyse the logical form of the Sorites Paradox and then show that Chisholm's Paradox exhibits (in all relevant aspects) the same formal structure.

One typical Sorites Paradox arises from employment of some vague predicate such as 'tall' or 'bald' and so on. Such a predicate clearly admits of degrees, e.g., 'John is tall' might be true to a higher degree than 'Peter is tall' is also true. The vagueness makes room for the idea according to which 'Peter is tall' might be true while 'John is taller than Peter' might also be true. Supposing that 'John is tall' is true, we may want to say about Peter, one centimetre shorter than John, that 'Peter is tall' can be true. And, about someone else, one centimetre shorter than Peter, 'he is tall' is true. Generalizing the same idea, if we truly say about some x that 'he is tall' is true of x , then we say about some y , one centimetre shorter than x , 'y is tall'. Of course, we can say about some z , one centimetre shorter than y , that 'z is tall' is also true. Absolutely nothing special needs to be added to the reasoning to lead us to say about someone 1.2 meters tall that 'he is tall', but, of course, this cannot be true. Thus, something must be wrong with respect to this way of understanding vague predicates.

Let us look more closely at the logical form of the reasoning: 'T' stands for the predicate 'is tall'. The sequence of letters $a, b, c \dots$ are names of individuals organized in such way that one individual who shows up sooner in the list is always one centimetre taller than your next closest neighbour and so on. Then, the paradox presents the following form:

$$\begin{array}{l} T a \\ T a \rightarrow T b \\ T b \rightarrow T c \\ \dots \\ \hline T n \end{array}$$

The logical form of the standard Sorites Paradox is quite simple. Starting with an object a that is correctly recognized as tall and applying the idea that the predicate 'is tall' is vague, then for any object b , one centimetre shorter than a , b must be recognized as tall as well. Applying the same idea

through all further steps of the reasoning – applying the predicate ‘is tall’ to the object in the consequent of each conditional one centimetre shorter than the object that shows up at the antecedent of the same conditional – we will reach one clearly short object and we will apply to it the predicate ‘is tall’. So, ‘Tn’ should be false, but our reasoning does not rule it out. Therefore, the paradox has been achieved.

The next move of Forbes’ strategy is argue to show that there is an S5-equivalent formulation of Chisholm’s Paradox which is formally analogous to the Sorites Paradox considered above. In order to establish such an analogy, Forbes needs to come up with a version of the paradox in which the significant intermediate steps look formally like $Ta \rightarrow Tb$; a *conditional* expressing a vague predicate. However, neither Chisholm’s own formulation nor the formulation considered above looks like that. As I have shown, the modal paradox looks like this:

$$\begin{array}{l} \Diamond \text{Mam}_0 \\ \Box (\text{Mam}_0 \rightarrow \Diamond \text{Mam}_1) \\ \dots \\ \Box (\text{Mam}_{n-1} \rightarrow \Diamond \text{Mam}_n) \\ \hline \Diamond \text{Mam}_n \end{array}$$

As might be clear, the present formulation of the paradox is not formally equivalent to the ordinary Sorites Paradox, which has been presented with a conditional such as $A \rightarrow B$ in the more relevant steps, whereas the current formulation is a quite different formula such as $\Box (A \rightarrow \Diamond B)$. Modal operators that appear in Chisholm’s Paradox make room for a formal disanalogy with the ordinary Sorites Paradox. However, Forbes noticed that there is an S5-equivalent to $\Box (A \rightarrow \Diamond B)$ which is plausibly rather closer to the conditional of the Sorites Paradox. Forbes has shown (and the reader can easily check it out by himself) that the formula $\Diamond A \rightarrow \Diamond B$ is S5-equivalent to $\Box (A \rightarrow \Diamond B)$. So, we may replace all steps of the form $\Box (A \rightarrow \Diamond B)$ with $\Diamond A \rightarrow \Diamond B$, which will lead us to a formulation for the paradox arguably analogous to the Sorites Paradox. Such a formulation would look like this:

$$\begin{array}{l} \Diamond \text{Mam}_0 \\ \Diamond \text{Mam}_0 \rightarrow \Diamond \text{Mam}_1 \end{array}$$

$$\dots$$

$$\Diamond \text{Mam}_{n-1} \rightarrow \Diamond \text{Mam}_n$$

$$\Diamond \text{Mam}_n$$

The latter formulation of the paradox is also S5-valid, even though the latter is, more plausibly, an analogous modal version of the Sorites Paradox. If that is the case, and Forbes has successfully shown that Chisholm's Paradox is actually an instance of the more familiar Sorites Paradox, then any good solution to the Sorites Paradox will hopefully be a successful solution to Chisholm's Paradox as well. Moreover, we have to concede at least one point to Forbes: the latter formulation is formally much closer to the logical form of the Sorites Paradox than the former.

Argument: the second stage – dealing with the Sorites Paradox

If Forbes has persuaded us about his proposal, then his next step to provide a solution to Chisholm's Paradox will be to come up with a plausible treatment of the ordinary Sorites Paradox. This section is reserved to introduce Forbes' strategy to deal with the Sorites Paradox, and then, in the next section, that strategy will be applied to his proposal concerning Chisholm's Paradox.

Forbes' strategy to deal with the Sorites Paradox is rather easy to be implemented, even though it implies a significant alteration in the core of first-order logic semantics. Actually, the main point of Forbes' strategy is to provide an alternative semantics to first-order logic which will end up rejecting *modus ponens* in vagueness contexts. In order to do that, he elects a semantics based upon many-valued models, i.e., a semantics to which a value corresponding to a real number between 1 (to maximal satisfaction) and 0 (minimal satisfaction) is attributed to each semantic structure. A logic with such a semantic treatment is often called '*many-valued logic*' or, occasionally, just '*fuzzy logic*', due to borderline cases of vagueness. For this logic, the following functions of degrees of truth redefine usual operators of classical logic:

$$v(\neg\phi): 1 - v(\phi)$$

$v(\phi \vee \psi)$: If $v(\phi) \geq v(\psi)$, then $v(\phi \vee \psi) = v(\phi)$. If $v(\psi) \geq v(\phi)$, then $v(\phi \vee \psi) = v(\psi)$.

$v(\phi \wedge \psi)$: If $v(\phi) \geq v(\psi)$, then $v(\phi \wedge \psi) = v(\psi)$. If $v(\psi) \geq v(\phi)$, then $v(\phi \wedge \psi) = v(\phi)$.

$v(\phi \rightarrow \psi)$: $1 - [v(\phi) - v(\psi)]$, if $v(\phi) > v(\psi)$.
1, otherwise.

What is in fact relevant for us here is the conditional, yet it might be useful to think a little about the other functions as well. The first one is quite clear and defines negation. If a formula p gets a degree of 0.7 true, then its negation will get a degree of 0.3. A disjunction with atomic formulae p (0.6 true) and q (0.2 true) will get the degree 0.6 for the molecular formula $p \vee q$. Similarly, the molecular conjunctive formula composed of $p \wedge q$ with degrees of 0.6 to p and 0.5 to q will hold 0.5 as its degree. It is worth noting that for any case of maximal and minimal semantic values (1 and 0), Forbes' proposal will always lead to exactly the same outputs as classical logic, i.e., any formula acknowledged as true in classical logic will be (in that context) acknowledged as true by Forbes' many-valued logic as well. This feature may easily be verified by the reader.

Conditional formulae, as I have said above, are the relevant ones for us and might be bit trickier, but even if that is so, it is not a special attribute of Forbes' proposal; functions of truth for conditionals are usually a little bit more complicated.

The degree of truth for a conditional is a function which is intended to reflect - in the degree of truth for the whole conditional - how much truer the antecedent is (compared to the consequent). In this case, the closer the gap between antecedent and consequent is to 1, the falser the conditional is. For example, if the antecedent is 0.8 and the consequent is 0.1, then the conditional will be falser than when the antecedent is 0.8 and the consequent is 0.4. In the former, the conditional is 0.3 and, in the latter, the conditional is 0.6. Thus, the latter is twice truer than the former. Considering the extreme case, when the antecedent goes with 1 and consequent goes with 0, the conditional will be wholly false.

We have just seen how Forbes' many-valued logic works, but how can it handle the Sorites Paradox? In order to explain Forbes' solution, we

must introduce one more important notion: *validity*. If a rule (an argumentative form) is valid, then for any application its conclusion never takes a degree of truth lower than the degree of truth of the lowest premise. In that sense, validity for this many-valued logic is a case of *degree-of-truth-preserving* rather than a case of *truth-preserving* as usual. Holding such a notion of validity for many-valued logic, it is quite easy to show the invalidity of *modus ponens*. Consider an instance A of *modus ponens*, A's first premise ϕ takes 0.7 and the second one $\phi \rightarrow \psi$ takes 0.9. Then, A's conclusion ψ takes 0.6. Thus, this instance of *modus ponens* must be invalidated and, then, *modus ponens* itself is shown to be an invalid rule for this many-valued logic. Since *modus ponens* is not allowed, a straightforward solution to the Sorites Paradox is made available.

1. Ta
2. Ta $\rightarrow$ Tb
3. Tb $\rightarrow$ Tc
- ...

4. Tn

If Ta takes a degree of truth (0.8) and Tb takes a slightly lower degree (0.7), then the degree of Ta $\rightarrow$ Tb is going to be (0.9). The conclusion, in this case, is Tb (0.7), which is lower than the degree of truth of the first premise Ta (0.8) and the second premise Ta $\rightarrow$ Tb (0.9). If this is correct, then the move to the conclusion is not allowed. The very same reasoning may be applied to each step of the Sorites Paradox in such a way that there will not be a paradox anymore. As I said, once we reject *modus ponens*, a solution to the Sorites Paradox is made available. Does the same happen with respect to Chisholm's Paradox? Let us look into this right now.

Argument: the third stage – extending the strategy to Chisholm's Paradox

This section focuses on extending the strategy of handling the ordinary Sorites Paradox to Chisholm's Paradox. In order to proceed with that extension, Forbes tries to establish an analogy between the two sorts of paradoxes, such an analogy being provided in both the formal and the philo-

sophical sense. From the formal point of view, Forbes must show that the paradoxes share the same logical form. *Prima facie*, this first task has already been done. Both paradoxes are formed by n steps of conditional formulae, where a predicate is attributed to some object, in the ordinary case the predicate ‘is tall’ and in the modal case ‘ a is possibly constituted of x ’. The modal operator does not seem to be enough to establish a disanalogy here, at least from the formal point of view. Nevertheless, from the philosophical point of view, things might get a little bit more complicated, since Forbes must be able to show (at least) that the *same kind* of vagueness phenomenon appears in both cases. Concerning the ordinary Sorites Paradox, the vagueness is related to the idea that we are not in a position to say about two individuals next to each other at the line of application of the predicate that ‘is tall’ is determinately applied to one but not to another. So, in the ordinary case, ‘is tall’ is the vague predicate. Thus, analogously, Chisholm’s Paradox is supposed to present a vague predicate as well. The best candidate is ‘ a could originally have been constituted of x ’, which is translated by Forbes’ counterpart theoretical account into ‘some of a ’s counterpart in a world w is originally constituted of x ’. Even though both the direct modal predicate and its counterpart theoretical translation are not obviously vague as their ordinary Sorites analogous is, they may be interpreted as so. In this case, there will be cases of indeterminacy concerning *de re* possibility and its counterpart reading. If the modal predicates are found to be analogous to ordinary vague predicates, then they might be satisfied in different degrees. And that is exactly what Forbes needs to extend his solution for the ordinary Sorites cases to his version of Chisholm’s Paradox.

Forbes suggests the following translating method for one of the intermediate conditionals of Chisholm’s Paradox into counterpart theoretical language. We should read ‘ $Cxyw$ ’ as ‘ x is a counterpart of y at w ’ and similar to what we have stipulated before ‘ $Mxyw$ ’ ‘ x is originally composed of y at w .’

$$\exists u \exists x \exists y (Cxa_u \wedge Cym_u \wedge Mxy_u) \rightarrow \exists v \exists x \exists y (Cxa_v \wedge Cym_v \wedge Mxy_v)$$

By this translation and supposing that counterparthood allows for degrees, then the degree to which x is a counterpart of a at u might be slightly higher than y is a counterpart of a at v . For instance, if x at u was

made of more of the wood of which a was actually made than y in v , then the antecedent of the conditional might be slightly truer than the consequent. Putting it straight, since counterparthood allows for degrees, one of the counterparts of a might be made of more of the wood that a was actually made of, which is sufficient to create a conditional wherein the antecedent is truer than consequent. As we have seen, that was exactly the motivation for the rejection of *modus ponens* in order to provide a solution to the ordinary Sorites Paradox. Therefore, Forbes argues, if the same phenomenon is happening with Chisholm's Paradox, then the same solution will be expected. Once *modus ponens* is rejected, every step of the Chisholm's Paradox is blocked. Thus, there will no longer be a paradox.

Salmon's solution

In this section, I will be presenting Salmon's solution to Chisholm's Paradox. As I have said, Salmon's solution is impressively simple and so it will not take us so long to present it. Actually, it is just based on the rejection of the modal principle axiom schema distinctive of **S4** modal system, which holds unrestrictedly that what is *possibly possible* is *possible*; or its equivalent necessity axiom, that holds that what is *necessary* is *necessarily necessary*. Using Kripke semantics, Salmon's solution is based on the idea that the accessibility relation between possible worlds is *intransitive*, which means that, supposing that R stands for the accessibility relation between possible worlds, in his framework w_0Rw_1 and w_1Rw_2 will not entail w_0Rw_2 . From Salmon's point of view, the fact that a sentence '*possibly possible* ϕ ' is true with respect to the actual world, for instance, does not entail that the sentence '*possible* ϕ ' is also true with respect to the actual world. In other words, the fact that there are some possible worlds *accessible relatively* to some possible worlds *accessible* from the actual world where ϕ is true does not entail that those worlds where ϕ is true are themselves *accessible relatively* to the actual world. Analogously, the dual necessity reasoning works as follows: the fact that a sentence '*necessary* ϕ ' is true with respect to the actual world does not entail that the iterated sentence '*necessarily necessary* ϕ ' is also true with respect to the actual world. To put it in slightly different

words, from the fact that ϕ is true with respect to every possible world accessible relatively to the actual world does not follow that with respect to any possible world accessible to those worlds accessible to the actual world, ϕ will also be true with respect to them. However, how can the restriction on modal logic systems settle the paradox? The solution follows straightforwardly from the restriction on the accessibility relation between possible worlds or, equivalently, from the rejection of the modal principle asserted by the axiom schema of **S4**. Looking at the derivation of the paradox given above, Salmon's solution just blocks all the S4 stage of the proof. Since S4 stage is not allowed, then no contradiction can be reached.

Despite being quite simple, Salmon's solution *rejects transitivity* on accessibility relation between possible worlds, which means that neither **S4** nor **S5** modal logic systems are adequate for modelling possibility and necessity. Maybe **B**, an alternative system to **S4**, or even **T**, a system weaker than **B** and **S4**, should be taken as adequate. One may think this is too high a price to pay.

Examining the alternatives

Salmon's proposal

In this section, I will be examining Salmon's transworld identity strategy. I will present two general objections that Salmon may face: one of them is related to the rejection of S5 modal logic, the other one related to ungrounded differences.

Rejection of S5

One may claim, against Salmon, that no system weaker than **S5** can accurately represent *metaphysical modality*. In order to defend his claim, he may invite us to think about different kinds of modality; the claim mentioned *modality* but did not say what kind of modality. So, let us think about some different kinds of modality to formulate the objection. Consider the proposition 'it is impossible to travel from Lisbon to Rio in less than one hour', what kind of modality would have it true? It does not seem to be true

if the *logical possibility* is taken into consideration. It seems obvious that it is *logically possible* to make a trip like that. Thus, we can conclude that *logical modality* does not make the proposition in case true. The *Metaphysical modality* will also not help. It is not (we hope) *metaphysically impossible* to make such a trip in less than one hour. But it seems to be quite appealing to say that there is a sense in which the proposition mentioned above is true and, in order to make room for this idea, we may want to recognize another kind of modality; we can call it ‘technological modality’. It is pretty obvious that it is *technologically impossible* to make that trip. Since we have recognized that proposition as true (under the technological modality reading), we might ask ourselves whether the proposition itself is necessarily true or just contingently true. If we say that it is necessarily true, then we are stating that it is necessarily impossible to travel from Lisbon to Rio in less than one hour. If that is the case, then **S5** modal system will correctly model technological modality, since under **S5** what is *impossible* is *necessarily impossible* and what is *possible* is *necessarily possible*. Nevertheless, it seems to be clear that if things had been different (if we had more advanced technology), then it would be possible to make that trip. So, despite being impossible to travel from Lisbon to Rio in less than one hour, it is not necessarily impossible, indeed it is just contingently impossible. The idea of *contingent impossibility* requires a system weaker than **S5**. Therefore, technological modality cannot be modelled by **S5**.

Proceeding with analogous reasoning about the logical possibility, one can ask if there is any proposition logically impossible that could have been possible? Supposing that ‘ $2+2=5$ ’ is logically impossible, could it have been possible? If things had been different in some way, would ‘ $2+2=5$ ’ be possible? It is arguable that the most plausible answer would be: no, there is no way in which ‘ $2+2=5$ ’ could *possibly* be true. So, if that is the case, then logical modality is arguably best represented by **S5** whereas technological modality would require a system weaker than **S5**.

The last step of the objection is to ask if the metaphysical modality is more similar to a technological modality or logical modality. Since metaphysical modality is supposed to be the broadest kind of modality, one argues, then metaphysical modality is likely to behave analogously to logical

modality. Hence, metaphysical modality is likely best represented by the **S5** modal system.

Ungrounded differences

As I showed during the presentation of the paradox, $\Diamond \text{Mam}_n$ is false with respect to the actual world (due to modal principle E). What does that mean? It means that there is no world possible with respect to the actual world where Mam_n is true. There is another proposition which Salmon recognizes as true at the actual world, a proposition whose modal operators are iterated as $\Diamond \Diamond \text{Mam}_n$. What does that mean? There is a world, w_1 , possible relative to the actual world, from which Mam_n is possible. So, there is a possible world, w_2 , possible with respect to w_1 where Mam_n is true. It is worth keeping in mind that, from Salmon's point of view, $\Diamond \Diamond \text{Mam}_n$ does not imply $\Diamond \text{Mam}_n$. By doing that he is able to coherently hold that it is impossible for a to be made of m_n , while holding that it is possibly possible to it for be made so. Since his models are not transitive, the fact that w_2 is possible relatively to w_1 , does not entail that w_2 is possible relative to the actual world.

It is arguable that there is something else that is true with respect to the actual world: some table or other could be made of m_n . So, $\Diamond \text{Mbm}_n$ does not seem to be ruled out by anything that has been said so far. If $\Diamond \text{Mbm}_n$ is actually true, then there is a world, w_3 , possible with respect to the actual world, where a table other than a is made of m_n (matter from which a could not have been made of). The important question now is: what grounds the difference between w_3 and w_2 ? Supposing that anything else is kept unchanged, the only difference between w_3 and w_2 is that w_3 is possible with respect to the actual world and w_2 is not. But why is that? Salmon should not appeal to accessibility relation in order to explain the difference since he would be begging the question. Furthermore, if the difference relies only on accessibility relation, it is really hard to show that the difference is grounded in intrinsic properties, which may suggest that the difference is just an ungrounded difference.

Forbes' proposal

In this section, I will be examining Forbes' counterpart theoretical strategy of dealing with Chisholm's paradox. I will be presenting basically two objections: first and more specifically, I will try to show that Forbes' many-valued logic does not validate an important metatheorem that we may not be in a position to give up on. Second and more generally, I will argue that the accessibility relation strategy may deal better with the notion of degree of *de re* possibility than Forbes' many-valued logic.

First objection

Forbes' logic does not validate *the deduction theorem*. What is the deduction theorem? I will briefly explain what the deduction theorem is and then I will show why Forbes' logic does not validate it. One way of presenting the deduction theorem is the following:

$$\Gamma, \varphi \models \psi \text{ if, and only if, } \Gamma \models \varphi \rightarrow \psi$$

φ and ψ represent any formula of the language and Γ represents a set of formulae (possibly empty). So, what the theorem states is that if the inference to the formula ψ from a set of formulae Γ and the formula φ is valid, then the inference to the conditional formula

$\varphi \rightarrow \psi$ from Γ is also valid. If the inference to the conditional

$\varphi \rightarrow \psi$ from a set of sentences Γ is valid, then the inference from Γ and φ to ψ is also valid. The right-left and the left-right directions give us the so-called 'deduction theorem'. What is important to be noticed here is that the deduction theorem establishes clear *parallelism* between validity and the material conditional. Intuitively, it reflects the idea according to which a conditional is *false* only when its antecedent is true and its consequent is false. And an inference is *invalid* only when there is a case where all its premises are true and its conclusion is false. Although we cannot go through the details concerning the importance of the deduction theorem, it is worth mentioning one of its more relevant features. The deduction theorem allows us to use an extremely important strategy of proof; namely, the conditional proof. Using the conditional proof, if we can prove, under the hypothesis that φ and a set Γ are true, that ψ , then we can prove $\varphi \rightarrow \psi$ from Γ .

This type of proof, among other things, is immensely useful to make proofs simpler and shorter.

The task of showing that Forbes' logic does not validate the deduction theorem is simple. As we have seen, the deduction theorem establishes the parallelism between validity and conditional, so, if we show that Forbes' logic does not reflect this *parallelism*, then we will have shown that Forbes' logic does not validate deduction theorem. Let us recall Forbes' definitions for validity and the truth function for conditionals. The truth function for *conditional* is the following:

$$v(\phi \rightarrow \psi): 1 - [v(\phi) - v(\psi)], \text{ if } v(\phi) > v(\psi). \\ 1 \text{ otherwise.}$$

And *validity* has been defined as follows:

A rule (an argumentative form) is valid iff for any application its conclusion never takes a degree of truth lower than the degree of truth for the lowest premise.

Let us consider again a case used in the argument for the Sorites Paradox. The argument is a *modus ponens* that, as we have seen, is invalidated in Forbes' logic. Supposing the following *modus ponens* with arbitrary values: $Ta \rightarrow Tb$, $Ta \models Tb$. Under the hypothesis that Ta takes the degree of truth (0.8) and Tb takes a slightly lower degree, let us say (0.7), the degree of $Ta \rightarrow Tb$ is going to be (0.9). The conclusion, then, is going to be Tb (0.7), which is lower than both the first premise Ta (0.8) and the second premise $Ta \rightarrow Tb$ (0.9). Therefore, this instance of *modus ponens* is not valid, since its conclusion reaches a degree of truth lower than its premises. As we have seen, Forbes gives up on *modus ponens* in order to solve both the Sorites Paradox and Chisholm's Paradox.

However, the deduction theorem allows us to infer from the very same *modus ponens* the formula $((Ta \rightarrow Tb) \wedge Ta) \rightarrow Tb$. Keeping the arbitrary values attributed before, we will conclude that this formula takes a degree of truth of 0.9, this time following the truth function for conditionals. What this reasoning shows us is that, even though Forbes' logic does not recognize an instance of *modus ponens* as valid, it attributes a degree of truth of 0.9 to its correspondent formula. Hence, Forbes' logic does not present the *parallel* treatment for validity and conditional that is required in

order to validate deduction theorem. Even if Forbes had had the intention to break this *parallel* treatment, one could not be in a position to give up on deduction theorem, he could prefer to keep it rather than accept Forbes' logic to solve the paradoxes considered here.

Degree of de re possibility

Forbes has replaced transworld identity of the standard Kripke semantics by counterparthood in his many-valued logic. As far as I can understand, the reason for doing so is to avoid a weird notion of degree of identity and to limit his theory for degree of *de re* possibility. Since counterparthood relation admits of degrees, an analysis for *de re* possibility based upon counterparthood does not give rise to additional problems. The question now is how can we understand the notion of degree of *de re* possibility based upon the counterparthood relation and, at the same time, how to analyse this notion in terms of degree of truth in such a way that allows us to extend the solution for the Sorites Paradox to Chisholm's Paradox? Unfortunately, Forbes does not provide a simple answer to this question. So, let us think about a conditional from Forbes' reformulation to Chisholm's Paradox, such as $\Diamond \text{Mam}_0 \rightarrow \Diamond \text{Mam}_1$, and let us suppose a degree of truth of (0.9) for the conditional. What does that mean? Well, following Forbes semantics, we have to say that the antecedent (0.8) is slightly truer than the consequent (0.7). But, why is the antecedent slightly truer than the consequent? The answer here must appeal to some notion related to the degree of *de re* possibility, something like *it is more possible for a to be made of m_0 than m_1* . How can we understand that? There are at least two ways to understand it: we may either introduce a notion of distance between possible worlds or an inner world notion of degree of truth. The latter seems to follow more naturally from Forbes' theory, although it might be quite implausible. Under this interpretation, we should say that, in the possible world where Mam_0 is 0.8 true, *a is wholly made of m_0* but in degree of truth of 0.8. If this interpretation is not incoherent, it is at least quite implausible. Even though we accept that *possibly being made of* is (in some sense) a predicate of degree, *being made of* does not seem to fashion degree in exactly the same way as *being*

tall. Thus, this interpretation is arguably not the best option. The former will introduce degree of truth over an additional notion of distance between possible worlds. In this case, despite '*a is wholly made of m_0* ' being fully true in some world and '*a is wholly made of m_1* ' being fully true in some other possible world, the difference in degree of truth between the antecedent and the consequent is explained by appealing to the notion according to which the possible world where *a is made of m_0* is closer to the actual world than the world where *a is made of m_1* . So, we keep the inner world notion of degree of truth and we explain degree of *de re* possibility in terms of the distance between possible worlds. This interpretation is more plausible and makes Forbes' theory more uniform. But, one may argue, it is just going in circles and begging the question. It explains degree of *de re* possibility in terms of the distance between possible worlds and then explains the notion of distance between possible worlds in terms of degree of *de re* possibility.

Furthermore, whether Forbes embraces one explanation or another, he will always have at least one more possible competitor; the accessibility explanation of degree of *de re* possibility. Under non-transitive systems for modal logic, the modal operator might be iterated as many times as necessary, which can be used to explain degree of *de re* possibility, e.g., $\Diamond M a m_n$ might be falser than $\Diamond\Diamond M a m_n$ and so on. In this sense, one can argue that there is a weaker sense of *de re* possibility according to which it is true that it is possibly possible for *a* to be made of m_n .

Summing up

From what we have seen so far, one obvious conclusion that we may reach is that both strategies, Salmon's transworld identity and Forbes' counterparthood, face important problems and that neither of them has provided a final solution to the paradox. Both have their formal and philosophical challenges. I do not think that any of what has been argued here undermines either one of the projects, but, at the same time, I hope that some of the arguments have shown that the two alternatives of invalidating strategies face important problems that have to be solved.

I am particularly inclined to think that Salmon's *transworld identity strategy* is the most conservative project. On the one hand, it retains most of the standard Kripke's semantics and gives up only on more marginal features of modal logic and modal metaphysics, i.e., *transitivity* of accessibility relation across possible worlds. Additionally, its formal and philosophical challenges seem to be easier to deal with. On the other hand, Forbes' *counterparthood strategy* changes the core of Kripke's semantics replacing transworld identity relation by counterparthood relation and also changing the semantics for first-order logic on the top of which modal logic is built substituting *bivalence* for a many-valued semantics. And as we have seen, it is not a simple task to provide a unified semantics for first-order logic and its extension for modal logic. Once again, I am not arguing that these problems undermine either one of the projects, what they may do is invite us to continue to work in both counterparthood strategy and transworld identity strategy in order to provide a successful invalidating solution for Chisholm's Paradox, if any solution of this kind is available. Such a kind of solution to the paradox is highly desirable since it would allow us to keep the modal principles E and T and, simultaneously, would allow us to avoid the paradox.

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