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Modeling deontic inconsistencies in moral dilemmas

Modelando inconsistências deônticas em dilemas morais

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ABSTRACT

Paraconsistent deontic logics have been proposed to deal with deontic paradoxes. We argue that these systems have been misdirected when used to solve deontic paradoxes not grounded on deontic inconsistency. Although the solutions are a formally satisfactory way out of the paradoxes, philosophically they are unsatisfactory. As an example, we discuss how some paraconsistent deontic logics have dealt with Chisholm's Paradox. A paraconsistent deontic logic should be used to solve paradoxes directly related to deontic inconsistency, as in the case of moral dilemmas. We show how some of these logics can model moral dilemmas while avoiding trivialization. We then consider the philosophical viability of their treatment of moral dilemmas. We argue that DPI is a system that models moral dilemmas in a philosophically satisfactory way. We offer an interpretation for this system, which reads the operator $\boxminus$ as formalizing non-dilemmatic situations. We discuss its philosophical consequences.

Keywords: deontic logic. paraconsistent logic. moral dilemmas. philosophy of logic.

RESUMO

Lógicas deônticas paraconsistentes foram propostas para lidar com paradoxos deônticos. Argumentamos que esses sistemas foram mal direcionados quando utilizados para resolver paradoxos deônticos que não surgem de inconsistências deônticas. Ainda que as soluções sejam um modo formalmente satisfatório de resolver os paradoxos, filosoficamente elas são insatisfatórias. Como exemplo, discutiremos como algumas lógicas deônticas paraconsistentes lidaram com o Paradoxo de Chisholm. Uma lógica deôntica paraconsistente deve ser utilizada para resolver paradoxos diretamente relacionados com inconsistência deôntica, como no caso de dilemas

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morais. Mostramos como algumas dessas lógicas podem modelar dilemas morais evitando trivialização. Consideramos, então, a viabilidade filosófica de seus tratamentos de dilemas morais. Argumentamos que DPI é um sistema que modela dilemas morais de uma maneira filosoficamente satisfatória. Oferecemos uma interpretação para o sistema, na qual lemos o operador $\boxminus$ como formalizando situações não-dilemáticas. Discutimos as consequências filosóficas dessa interpretação.

Palavras-chave: lógica deôntica. lógica paraconsistente. dilemas morais. filosofia da lógica.

Introduction

When dealing with deontic logics, we are usually interested in notions such as *obligation*, *permission*, *prohibition*, and others. Since the beginning of the study of such area in the XX century, the path to a logical account of moral systems was to assume that they are well described by a (normal) modal logic system designed to deal with alethic notions (Hilpinen and McNamara, 2013).

Trying to accommodate deontic concepts in systems designed to deal with alethic modality has, however, caused a lot of problems. Not by accident, deontic logic has been plagued by paradoxes and pragmatic oddities[3] since it became an area by itself after the works of G. H. von Wright (1951). This is, perhaps, because some presuppositions do not account for the possibility of properties intrinsic to moral systems, such as violations of obligations, the possibility of conflicting obligations, revising principles, adjustments, amendments, and so many others.

Many deontic logicians tried to circumvent the problems arising in deontic logics in many different ways. These include assuming new principles so that (K) and (D) do not determine the deonticity of the logic in some cases, adding a new implication in

[3] We mention the term *pragmatic oddity* which is used by Henry Prakken and Marek Sergot (1996). This term refers to principles that are logically acceptable, but when read as a deontic principle, may cause some oddity to happen. One such case is Ross' paradox (Ross, A., 1944, p. 41): "One ought to send a letter. If one ought to send a letter, then one ought to either send a letter or to burn it. Therefore, one ought to either send a letter or to burn it.". Notice that when formalizing such a case one does not get a logical contradiction, triviality, or explosion. However, it is still odd to think that one can derive the obligation to burn the letter or to send it *from* the obligation to send it.

order to deal with paradoxes such as Chisholm's in others (Lewis, 1974; Van Fraassen, 1973; Chellas, 1975), or perhaps even weakening the systems, so to reject one or more of the Standard Deontic Logics principles, as per the position defended by Ruth Barcan Marcus (1980). The latter, not only rejects this principle, but defends that conflicting obligations do in fact exist and that our deontic systems should be able to accept them without trivialization, and without needing to revise the whole system.

Following that approach, Newton da Costa and Walter Carnielli (1986) proposed a system that would solve that burden: if having conflicting obligations, i.e., if having the obligation to do A and the obligation to do $\neg A$, is the problem, we then should add the axioms of deontic logics to a system that accepts such a scenario without trivialization. Thus were born the paraconsistent deontic logics. In a similar approach, the work was developed by Marcelo Coniglio and Newton M. Peron (Coniglio, 2009; Peron and Coniglio, 2008; Coniglio and Peron, 2009), now using *LFI*s and PI as the base logics[4].

The formal systems they proposed are interesting and can be put to good use. But that is not what happened so far. What we see lacking in this literature is a proper philosophical motivation in favor of these systems. That is what we seek to do in this article. We have three complementary objectives: (I) We will argue that the systems proposed by Coniglio and Peron were used to deal with a deontic paradox they are not well-equipped to deal with; (II) This criticism is complemented by our proposal of redirecting these logics towards deontic paradoxes that they can solve in illuminating ways. Here, we will discuss moral dilemmas; (III) We will show how one of the systems proposed by Coniglio and Peron offers an interesting modeling of moral dilemmas. By proposing a scenario where deontic obligation is understood as moral obligation, we offer an interpretation of the consistency symbol proposed in the works of Peron and

[4] We also acknowledge that other researchers not mentioned above have worked similarly on questions about paraconsistent deontic logics, but in very different approaches. For example, Casey McGinnis (2007) bases his systems on Priest's Logic of Paradox, while Mathieu Beirlaen and Christian Straßer (2011), on Baten's adaptive logic.

Coniglio and discuss some philosophical implications.

The article is divided into five sections and one appendix. We first present a formulation of SDL, together with a discussion of some of the problems it bears (Section 1). Then we briefly retake the paraconsistent approach to deontic logics presented by Coniglio and Peron, presenting systems DPI, $SDPI$, $DmbC$, $SDmbC$ and $BDmbC$ (Section 2). We argue there that their treatment of Chisholm's Paradox, despite formally working, is philosophically unsatisfactory. After this criticism, we use the systems presented to deal with moral dilemmas (Section 3). We start by discussing why we think that they are well-equipped to deal with this specific deontic problem. We then show how they can model moral dilemmas without getting deontic trivialization. After that, we take a philosophical look at how these systems model moral dilemmas (Section 4). We argue that DPI gives us the best philosophical treatment of moral dilemmas. We give a philosophical interpretation for DPI's modeling of moral dilemmas. We, then, discuss some philosophical implications of the resulting interpreted system. We conclude by highlighting future work (Section 5). The appendix provides a complete description of DPI and a completeness proof.

We intend to develop the ideas presented in this paper further in future works, both in the philosophical and logical aspects. This is a first glance at what could be a future development of paraconsistent deontic logics.

1 A problem in Standard Deontic Logic

We start by giving a notion of SDL:

Definition 1.1. Let $\Sigma = \{\wedge, \neg, \rightarrow, \vee, \mathrm{O}\}$ and For_{Σ} be the set of formulas over Σ. The logic SDL is the logic characterized by all axioms of the propositional classical logic plus

(OK) $\mathrm{O}(\alpha \rightarrow \beta) \rightarrow (\mathrm{O}\alpha \rightarrow \mathrm{O}\beta)$

(DD) $\mathrm{O}\alpha \rightarrow \mathrm{P}\alpha$, where $\mathrm{P}\alpha := \neg\mathrm{O}\neg\alpha$

(ONec) If $\vdash \alpha$, then $\vdash \mathrm{O}\alpha$

The deduction rules are Modus Ponens and Uniform Substitution.

In simpler terms, SDL includes all the axioms for the logic K, while adding axiom DD which states that whatever is obligatory is also permitted.

Alternatively to stating DD, if our language has $\perp$, we could state axiom OD:

(OD) $\neg\mathrm{O}\perp$

which states that there are no contradictory obligations.

Semantically, what both of these axioms imply is that whenever we have a model that characterizes such a logic, its accessibility relation must be serial, i.e., it cannot have worlds that are dead-ends[5].

SDL is notably prone to many paradoxes, three of them are presented below[6]:

Ross' paradox $\mathrm{O}\alpha \rightarrow \mathrm{O}(\alpha \vee \beta)$

Good Samaritan $\mathrm{O}(\alpha \wedge \beta) \rightarrow \mathrm{O}\beta$

The gentle murderer $\mathrm{O}\neg\alpha \wedge (\alpha \rightarrow \mathrm{O}\beta) \wedge \vdash (\beta \rightarrow \alpha) \wedge \alpha \vdash \perp$[7]

Many of these paradoxes have been extensively studied in the literature, some of them entailing contradictory obligations, while in some cases, they are considered paradoxes because of their characteristic to derive some pragmatic oddities. An

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[5] Let W be a set of worlds and R a binary relation in W. We say that R is serial if and only if for any world $w \in W$, there is a world $w^{'} \in W$ such that $wRw^{'}$. Semantically, a sentence $\mathrm{O}\alpha$ is true in w whenever α is true in every $w^{'}$ such that $wRw^{'}$. if the relation is allowed not to be serial, then there is a world w which does not access any other world. But that means that $\mathrm{O}\alpha$ is true for every $\alpha \in \Sigma$, in particular, also $\mathrm{O}\neg\alpha$ is true. But then DD would be false.

[6] An extensive list of paradoxes can be surveyed throughout the literature, a good number of them are found in (Hilpinen and McNamara, 2013) and in (Meyer, Dignum, and Wieringa, 1994).

[7] Notice that the third conjunct here is taken to be a theorem, since it is read as "If one murders gently, then one is a murder". From this assumption, one can derive $\mathrm{O}\beta \rightarrow \mathrm{O}\alpha$, making the contradiction possible.

example of pragmatic oddity is the interpretation of Ross' paradox (Ross, A., 1944) usually formulated as "One ought to send a letter" which implies "One ought to send a letter or burn it". Another is the Good Samaritan's paradox (Åqvist, 1967), which can be interpreted as "It ought to be the case that you help A and A has been robbed"[8], from which we can infer "it ought to be the case that A is robbed". As an example of entailing a logical contradiction, the last cited paradox (Forrester, 1984) often assumes the following two premises "A is obliged not to kill" and "If A kills, then they must do it gently". It is a fact that "If A kills gently, then A is a murderer". Finally, we assume that, unfortunately, "A kills someone". In SDL, this set of sentences implies a contradiction.

Another paradox that is derivable in SDL is the famous Chisholm's Paradox[9] (Chisholm, 1963, pp. 34-35), which was originally formulated as follows:

1. It ought to be that a certain man goes to help his neighbours.
2. It ought to be that if he goes he tells them he is coming.
3. If he does not go, he ought not to tell them he is coming.
4. He does not go.

Attempts to solve this paradox have shown that there is difficulty in formalizing it satisfactorily. A few ways of formalizing the second premise might result in an interdependent set of formulas, while attempting to preserve independence makes this set of sentences inconsistent (Åqvist, 1967). Ideally, independence should be preserved between a set of sentences, which results in the following formalization being the standard for this paradox:

[8] We omit the parenthesis for the sake of reading it more naturally. This sentence should, however, be read "It ought to be the case that (A is helped by you and A is robbed)". We also do not reproduce the exact phrasing used by Åqvist.
[9] The gentle murderer paradox is a particular case of a contrary-to-duty obligation paradox, of which Chisholm's Paradox is a more general example.

1. $O\alpha$

2. $O(\alpha \to \beta)$

3. $\neg\alpha \to O\neg\beta$

4. $\neg\alpha$

Notice how axiom OK and Modus Ponens deduce $O\beta$ from 1. and 2., while $O\neg\beta$ is deduced from 3. and 4. by Modus Ponens. Further, since aggregation is a theorem of SDL, we deduce $O(\alpha \wedge \neg\alpha)$, which is equivalent to $O\perp$. Given that $\neg O\perp$ is an instance of the (OD) axiom, we have a contradiction.

Besides this derivation, we can also have readings of the paradox that would entail pragmatic oddities. In this sense, it is interesting to see the approach proposed in the works by Henry Prakken and Marek Sergot (1996; 1997), who add the constraint that no pragmatic oddity should arise, thus proposing a dyadic obligation operator in order to deal with this paradox. This route is one of the most interesting ones, though it relies on neighborhood semantics in order to describe the behavior of the obligatory operator.[10]

One reason given to explain why SDL entails these so-called strange conclusions is that its modal axioms are too encompassing. A great example of such a thesis is given by Marcus (1980), where she claims that we should assume weaker axioms for our deontic logic, in other words, to allow conflicting obligations without collapsing.

For Marcus, an adequate deontic system should not validate, for example, O-aggregation:

$$\vdash O\alpha \wedge O\beta \to O(\alpha \wedge \beta)$$

A good reason for avoiding this principle is justified by an agent being able to be obligated to do α and also obligated to do $\neg\alpha$, for example when the actions are

[10] The jump from possible world semantics to neighborhood semantics to deal with such systems can be seen also in (Lewis, 1974), (Chellas, 1980) and (Prakken and Sergot, 1997).

not being considered at the same time or in the same context. However, since $\alpha \wedge \neg\alpha$ is a logical contradiction, then it is an impossible action, hence there cannot be cases in which the same agent is obligated to carry out the action $\alpha \wedge \neg\alpha$.

These too-encompassing principles could be substituted for weaker versions or taken out altogether from our system of rules in order to block a few of these odd conclusions. This is part of the approach we advocate in this paper, but we would also like to *add* something else, namely, a purely deontic principle that treats consistency in the modal level instead of the propositional level.

Having looked at the approaches that take classical logic as a basis for deontic logic, we can see that paradoxes abound in this scenario. Trying to fix the problem by not changing the basic system and only adding more axioms to it seems to preserve some of the paradoxes. Our option now is to change something. Few authors propose, for example, changes that deal with semantic notions as seen in (Lewis, 1974). Although this approach is interesting, that could be a topic for a paper by itself. We will focus, instead, on the tradition that stems from the work on deontic logics carried out by (Costa and Carnielli, 1986), which propose that a natural way to deal with conflicting obligations in a logical system is to assume paraconsistency.

2 The Paraconsistent Solution

Both Chisholm's Paradox and Gentle Murderer's Paradox result in the trivialization of the system because of the principle of explosion. One direct way of avoiding this trivialization is to adopt a base system that would not explode in face of a contradiction. This can and has been done so by changing the base logic to a paraconsistent one.

The first work to propose such a change is (Costa and Carnielli, 1986), which based their system on da Costa's $\mathbf{C}_1$. Later, Ângela Maria Paiva Cruz (2005) advanced a few steps by adding a dyadic deontic system, which is a really interesting approach to the matter, since it combines the virtues of dyadic deontic systems with a change in paradigm by changing the base logic. A deeper study of alike systems was then developed by Coniglio and Peron (Peron and Coniglio, 2008; Coniglio and Peron, 2009; Coniglio, 2009), who studied a great number of systems, mainly minimalist *LFI*s[11] (Carnielli, Coniglio, and Marcos, 2007) and Diderik Batens' PI[12] (1980; 1980) as the base logics for their work.

Regarding the minimal *LFI*s, these are logics whose signature has a unary operator $\circ$ and that are not generally explosive[13], which means that there there are two formulas α, β such that

$$\alpha, \neg\alpha \nVdash \beta$$

and also there are two formulas φ, ψ such that:

$$\circ\varphi, \varphi \nVdash \psi$$

$$\circ\varphi, \neg\varphi \nVdash \psi$$

and also satisfy weak explosion, namely, for all formulas α, β:

$$\textbf{(WExp)}\, \circ\alpha, \alpha, \neg\alpha \vdash \beta$$

The operator $\circ$ is generally interpreted as a consistency operator, being possible to recover classicality inside of them, assuming every formula is consistent, i.e., that $\circ\alpha$ is an axiom. Such systems are paraconsistent because they do not validate the

[11] *LFI* is an acronym for *Logic of Formal Inconsistency*.
[12] This logic has been renamed CLuN in more recent works, and this has been the name adopted by the community, since its acronym for Classical Logic glutty with respect to Negation.
[13] The general definition of *LFI*s is based on the consistent set p, which are the set of all formulas solely dependent on the propositional variable p and does not satisfy explosion for any formula in the set of formulas.

principle of explosion.

Advancing on the discussion, in their work from 2009, Coniglio and Peron named their systems *Logics of Deontic Inconsistency* (*LDI*s), in reference to the *LFI*s, but also establishing a few conditions for a logic to be called so. In short, analogous to the definition of *LFI*s, a logic is an *LDI* if, given a set $\boxminus p$ of deontic formulas based exclusively on the propositional variable p, there are formulas $\varphi, \varphi', \psi, \psi'$ such that:

$$\boxminus\varphi, O\varphi \nVdash O\psi$$

$$\boxminus\varphi, O\neg\varphi \nVdash O\psi$$

and it also satisfies deontic weak explosion, namely, for all formulas α, β:

(OWExp) $\boxminus\alpha, O\alpha, O\neg\alpha \vdash O\beta$

Whenever $\boxminus\alpha$ is a singleton, we denote it $\boxminus\alpha$ and it is interpreted as the *deontic consistency* operator.

We find this definition quite insightful. It fulfills exactly what we thought should be a basic condition for a *Logic of Deontic Inconsistency* to be called so, that is, that of analyzing consistency at the modal level. In previous works, consistency has been defined at the propositional level, and it was by accident that modal behavior of consistency was achieved. By defining an additional notion of consistency, the system gains an additional tool when modeling the complex phenomena we find in the deontic realm.

Definition 2.1. Let $\Sigma = \{\wedge, \vee, \rightarrow, \neg, O, \circ, \boxminus\}$ to be a signature. We take $For(\Sigma)$ to be the sets of formulas over Σ. Let $\alpha, \beta \in For(\Sigma)$ and consider the following axioms:

(EM) $\alpha \vee \neg\alpha$

(bc) $\circ\alpha \rightarrow (\alpha \rightarrow (\neg\alpha \rightarrow \beta))$

(O-K) $O(\alpha \to \beta) \to (O\alpha \to O\beta)$

(O-E)$^\circ$ $O\bot_\alpha \to \bot_\alpha$, where $\bot_\alpha := (\alpha \wedge \neg\alpha) \wedge \circ\alpha$

(O-E) $O(\alpha \wedge \neg\alpha) \to \bot_\alpha$

(Dbc1) $\boxminus\alpha \to (O\alpha \to (O\neg\alpha \to O\gamma))$ for all $\gamma \in For(\Sigma)$

(SDbc1) $\boxminus\alpha \to (O\alpha \to (O\neg\alpha \to \gamma))$ for all $\gamma \in For(\Sigma)$

(O-Nec) If $\vdash \alpha$, then $\vdash O\alpha$

and Modus Ponens and Uniform Substitution as inference rules.

The logic DPI is defined over $For(\Sigma \setminus \{\circ\})$, satisfying the axioms of CPL^{+}[14] plus **(EM)**, **(O-K)**, **(Dbc1)**[15] and **(O-Nec)**. The logic $DmbC$ is defined over $For(\Sigma \setminus \{\boxminus\})$, satisfying the axioms of CPL^{+} plus **(EM)**, **(bc)**, **(O-K)**, **(O-E)**$^\circ$ and **(O-Nec)**.

The logic $SDPI$ is obtained from DPI by adding **(O-E)** and substituting **(SDbc1)** for **(Dbc1)**. The logic $SDmbC$ is obtained from $DmbC$ by assuming **(O-E)**$^\circ$ in place of **(O-E)**.

We can also define the system $BDmbC$:

Definition 2.2. Let $\Sigma' = (\Sigma \setminus \{\boxminus\}) \cup \{O\}$ and $For(\Sigma')$ to be the set of formulas over Σ'. Then $BDmbC$ is the logic obtained such that O satisfies all axioms for $DmbC$, O satisfies all axioms for $SDmbC$, and we add one bridge axiom:

$$\textbf{(BA)}\ O\alpha \to O\alpha$$

The systems above presented comprise a good number of systems studied by Marcelo Coniglio and Newton Peron between 2007 and 2009 (Coniglio and Peron, 2009;

[14] CPL$^+$ is the system comprised of axioms 1.-9. of definition A.1 in the appendix, with modus ponens and uniform substitution as rules of inference.
[15] We write **(Dbc1)** for the version of **(OWExp)** written as an implication, to agree on the axioms presented in (Coniglio, 2009) and (Peron and Coniglio, 2008).

Peron and Coniglio, 2008; Coniglio, 2009). Another system mentioned by the authors is the system $\mathbf{C}_1^D$, which will not be explicitly mentioned here, despite being a historically important system, for brevity reasons[16].

The only paradox analyzed in the aforementioned works is Chisholm's Paradox, hence why we decided to give special attention to it when presenting paradoxes. Another point of interest is that they present the version of the paradox in (Åqvist, 2002), which is a negated version of the paradox. Here we opt to present Chisholm's original formulation. As seen in (Prakken and Sergot, 1996), the different formulations of the contrary-to-duty paradoxes do not significantly alter the results, unless they explicitly add another dimension to the problem, such as an agent, time, defeasible obligations, and so on.

In (Coniglio and Peron, 2009), the authors use the *LFI*s' characteristic of being able to recover classical negation, by means of defining it as $\sim\alpha \coloneqq \alpha \rightarrow \bot_\alpha$. This allows the authors to specify three distinct kinds of prohibition for the system $BDmbC$, $O\neg\alpha$, $O{\sim}\alpha$, and $O\neg\alpha$. For $DmbC$ only the first two can be distinguished. The systems $SDmbC$ and DPI have only the first kind of prohibition. The system $SDPI$ behaves as $DmbC$. Finally, they also present a few formulations of Chisholm's set of formalized sentences, resulting in many different formulations of the paradox.

The common results for these systems seem to be of two kinds. First, for some sets of sentences, the systems proposed avoid dependency. This comes from the fact that weak negation do not validate some of the usual axioms, which is the reason why some sentences are dependent in formalizations of the paradox in the classical logic context. Second, the systems do not trivialize under conflicting obligations, for those sets of sentences which are independent and where conflicting obligations occur. The authors follow the argument in (Åqvist, 1967) for independency of the set of sentences

[16] One more reason to not explicitly talk about this system is that the criticisms we draw and suggestions we propose can easily also be made to $\mathbf{C}_1^D$, since it is an extension of $DmbC$. In fact, the extension of mbC named $Cila$ is known to be equivalent to $\mathbf{C}_1$ (Carnielli, Coniglio, and Marcos, 2007; Carnielli and Coniglio, 2016).

of the paradox, so for each system, only the sets of sentences that are independent were studied.

DmbC, *DPI* and *SDPI* allow many different ways of formalizing the paradox, validating each of the obligations in some cases, both in others, and neither in others[17], with the observation that *DmbC* and *SDPI* behave similarly, *mutatis mutandis*. *SDmbC* on the other hand is only able to generate the positive obligation. *BDmbC* then presents an even larger set of solutions than *DmbC* and *SDPI*, because the sets can be formalized with two distinct types of obligations.

Although the plethora of answers and possibilities of formalization seems fascinating at first, none of them seems to be a fitting answer to the paradox. Avoiding the trivialization of the system is not enough to give a *philosophically* satisfactory solution. A philosophically satisfactory solution to a deontic paradox not only avoids a trivialization of the system but also does it in a way that explains what we got wrong about the scenario in the first place. A good solution illuminates our understanding of deontic concepts. This idea can be traced back to (Åqvist, 1967), when trying to find solutions for contrary-to-duty paradoxes, in which beyond the fact that the solution should be non-redundant and satisfiable, there is a third requirement that it should explain what sense of obligation is being used in the formalizations.

Here lies the problem with *LDI*s' treatment of Chisholm's Paradox: *LDI*s provide a formal solution to the paradox, but they focus on aspects of the formalization that are *not* where the central problem with Chisholm's Paradox lies. Chisholm's Paradox is essentially a problem about the relation between duties and conditionals (Hilpinen and McNamara, 2013). A good solution to it not only avoids the paradox but shows what we got wrong about this relation. The systems analyzed, however, deal with the paradox by avoiding explosion without explaining what obligation or negation are

[17] One can see all the formalizations in detail in section 3 of (Coniglio and Peron, 2009). There are collectively 16 possible sets of sentences that formalize Chisholm's Paradox. Analyzing each one would take us far from our objective here, so we opted for a general overview.

accounted for in these scenarios. Their solution, therefore, does not address the central philosophical question of this paradox we are trying to deal with.

We can see that Coniglio and Peron's solution to the Chisholm's Paradox is philosophically inadequate by paying attention to the philosophical motivations they give in favor of making a distinction between more than one sense of prohibition. They gesture towards Ross's distinction (Ross, W. D., 1930) between *prima facie* obligations and all-things-considered obligations (Coniglio and Peron, 2009, p. 319). For Ross, we have *prima facie* obligations that together produce an all-things-considered obligation. To illustrate the distinction, imagine a man at your door who wants to murder your friend. You have a *prima facie* obligation to not lie to him, given that lying is wrong. You also have a *prima facie* obligation to save your friend from mortal danger. The two *prima facie* obligations conflicted, but we may assume that the second obligation is stronger. Hence, you have the all-things-considered obligation to lie to the man in order to save your friend's life.

Despite Conigilio and Peron's system being able to model Ross's distinction to some degree, this distinction does not help us deal with Chisholm's Paradox. Every obligation involved in Chisholm's Paradox is an all-things-considered obligation. Hence, Ross's distinction between two kinds of obligations does not help with the paradox. Therefore, Conigilio and Peron's philosophical motivation for their system is at odds with their formal solution to Chisholm's Paradox. This is enough to show that their solution is philosophically unsatisfactory to deal with this paradox.

3 Moral Dilemmas and *LDI*s

So far, we have been criticizing *LDI*s. In this section, we start our positive contribution to this tradition of paraconsistent deontic logics. We will present a deontic problem, moral dilemmas, and show how the systems presented can model it without trivializing. In this section, we will be concerned with the formal aspect of modeling moral dilemmas. In the next section, we will focus on the philosophical aspect.

Before discussing moral dilemmas, we should explain why we chose to deal with this deontic problem. We took as a starting point what *LDI*s are good at. In them, we have the notion of deontic consistency represented by the $\circ$. Given that, we should use *LDI*s to deal with deontic paradoxes where the paradox is grounded on inconsistency between duties. There is a sense in which all deontic paradoxes are cases of inconsistency between duties (given that we arrive at an inconsistency). But in many cases, it is not an inconsistency between duties that originates the problem. For instance, in Chisholm's Paradox, we got an inconsistency because of a bad understanding of conditionals. We are interested now in a paradox that originates because of an inconsistency between duties. *LDI*s are able to model such paradoxes in illuminating ways. To show that, we will focus on the paradoxes arising from moral dilemmas.

Moral dilemmas are scenarios where one agent ought to do one action, ought to do another, but is unable to do both together. To logically model this, we will need to add another modality to a deontic logic, thus making it a bimodal logic. We assume a logic with two operators, O and $\Box$. The first represents moral obligations, and the second represents necessity. In other words, given a proposition A, $\mathrm{O}A$ means "it ought to (morally) be the case that A"[18] and $\Box A$ means "A is necessary". We do not delve into agency for the sake of simplicity and assume that we are talking about the same agent when mentioning a modal sentence in the system. We can thus work with an extended signature $\Sigma' = \Sigma \cup \{\Box\}$, where $\Box$ satisfies all axioms of logic K. There are two plausible mixed axioms for such a bimodal logic:

[18] We do not consider that there is a great advantage here on differentiating between *ought-to-be* cases and *ought-to-do* cases. We understand that the reasons the paradox arises in an *ought-to-be* formulation of the paradox are sufficiently close to the reasons that it would arise in an *ought-to-do* formulation. We used the standard definition of moral dilemmas in terms of *ought-to-do*, but we could also formulate it using *ought-to-be*: a moral dilemma is a situation where (i) it ought to be the case that A, (ii) it ought to be the case that B, (iii) the agent is unable to make the case that A and B.

OC $(OA \wedge \Box(A \rightarrow B)) \rightarrow OB$[19]

Kant's Law $O\alpha \rightarrow \Diamond\alpha$, where $\Diamond := \neg\Box\neg$.

Kant's Law is pretty straightforward to understand, being an attempt to formalize the Kantian principle of ought-implies-can. If you have a duty, it is possible for you to follow this duty.

OC, on the other hand, needs a little more work to be justified, but the rationale in its favor is very persuasive. If one has an obligation, and fulfilling it necessarily implies something else, one also has the obligation of doing this something else. For instance, if you have the obligation to save a drowning child and saving her necessarily implies getting your shoes wet, then you have the obligation to get your shoes wet.[20]

With this system, we can give a more formal definition of moral dilemma: A moral dilemma is a situation where $OA \wedge OB \wedge \neg\Diamond(A \wedge B)$ holds. If there really are moral dilemmas is an open philosophical question. On one side, we have dilemma realists, who believe that there are moral dilemmas (Williams, 1965; Van Fraassen, 1973; Bohse, 2005; Weber, 2007; Ribeiro Teles, 2021). On the other side, there are dilemma anti-realists, who argue that there are no moral dilemmas (Conee, 1982; Zimmerman, 1987; Merlussi, 2013). Usually, dilemma anti-realists argue for their position by showing that the definition of moral dilemma, together with a couple of plausible deontic principles, leads to a contradiction. To see this in a specific case, consider the following case of a (putative) moral dilemma:

Sophie's Dilemma: Sophie is a prisoner in a Nazi concentration camp. She is there with her son and her daughter. The boy and the girl are scheduled to be executed,

[19] This principle can be seen in (Prior, 1958, p.135), referring to a definition found in (Anderson and Moore, 1957). This principle was also informally presented in (Van Fraassen, 1973, p. 12) and (Lemmon, 1965, p. 40). We also acknowledge that similar principles, although not the same, have been analyzed, for example, in (Hintikka, 1971, pp.79-82). The name (OC) was given by us and stands for *obligation of the consequent*. We also acknowledge that this shortform has also been used for conditional obligations, as seen in (Chellas, 1975). We understand that no confusion may arise since our use is quite distinct.

[20] Ross's Paradox gives us a reason to reject or revise OC, but we will not discuss this here.

> but the guards decide to give Sophie a choice: she can save one of them from execution. If she does not choose, both will die as planned. She decides to save the son. The daughter is murdered.

This case seems to satisfy our definition of a moral dilemma. Where S is the action of saving her son and D is the action of saving her daughter, it is true of Sophie that $\mathrm{O}S \wedge \mathrm{O}D \wedge \neg\Diamond(S \wedge D)$. From this, the anti-realist gets a contradiction. There is more than one way to derive it.[21] We will present two. First, using OC, we can derive both a deontic inconsistency and a contradiction:

1	$\mathrm{O}S \wedge \mathrm{O}D \wedge \neg\Diamond(S \wedge D)$	Moral Dilemma
2	$\mathrm{O}S$	1
3	$\mathrm{O}D$	1
4	$\neg\Diamond(S \wedge D)$	1
5	$\Box(S \rightarrow \neg D)$	4
6	$\mathrm{O}\neg D$	2, 5 and OC
7	$\mathrm{O}D \wedge \mathrm{O}\neg D$	3 and 6
8	$\mathrm{O}(D \wedge \neg D)$	7 and O-Aggregation
9	$\mathrm{O}\bot$	8
10	$\neg\mathrm{O}\neg D$	3 and DD
11	$(\neg\mathrm{O}\neg D) \wedge \mathrm{O}\neg D$	10 and 6
12	$\bot$	11

We can also get the contradiction with Kant's Law:

[21] In our presentation of the anti-realist's arguments, we are broadly following (Merlussi, 2013).

1	$\mathrm{O}S \wedge \mathrm{O}D \wedge \neg\Diamond(S \wedge D)$	Moral Dilemma
2	$\mathrm{O}S$	1
3	$\mathrm{O}D$	1
4	$\neg\Diamond(S \wedge D)$	1
5	$\mathrm{O}(S \wedge D)$	2, 3 and O-Aggregation
6	$\Diamond(S \wedge D)$	5 and Kant's Law
7	$(\neg\Diamond(S \wedge D)) \wedge \Diamond(S \wedge D))$	4 and 6
8	$\perp$	7

We have, then, two sets of principles that seem to be incompatible with the existence of moral dilemmas: (i) DD and OC and (ii) O-Aggregation and Kant's Law. Realists usually have to deny one principle of each group to accept the existence of dilemmas.

If we want the systems presented above to model moral dilemmas, we face a problem right away: most of them were presented as monomodal logics, having only O as its modal operator. The exception would be $BDmbC$, which has two obligation operators, so it does not help our cause either. We understand, however, that there are reasons that allow us to treat moral dilemmas even in these monomodal systems. First, the problem of consistency presented by moral dilemmas concerns the obligations, not the possibilities in place. The crux of the matter is conflicting obligations, not conflicting possibilities. Second, our solution could be straightforwardly extended to systems that are multimodal and have at least one alethic operator[22]. So assume that we should have a moral dilemma whenever it occurs that $\mathrm{O}A$ and $\mathrm{O}\neg A$ both occur in a deduction.

In $BDmbC$ it should be noted that a dilemma can occur having one of the

[22] If any of the systems presented in the previous section satisfies **OC**, step 5 of the deduction is blocked, since $\neg(\alpha \wedge \beta) \to (\alpha \to \neg\beta)$ does not hold in any of those systems. If they satisfy Kant's Law, then the last step of the proof is blocked, unless $\Diamond(S \wedge D)$ is a consistent formula. Notice that the contradiction in the last case happens only because there were conflicting obligations in place. So despite not being the final product, the problem still relies on allowing conflicting obligations to occur.

conjuncts as a weak obligation, for example $\mathrm{O}A$ and the other, as a strong obligation, $\mathsf{O}A$. If this occurs, or if both are weak obligations, then such a situation should not pose a problem to the system. However, if both are strong obligations, then since O satisfies ($\mathbf{O}-K$), then we deduce $\mathsf{O}\perp$ and we have explosion. The logic $SDmbC$ behaves as in the last case.

If we are working in $DmbC$, this should not pose, *prima facie*, a problem for this system. It does not satisfy deontic explosion, so trivialization does not occur when both $\mathsf{O}A$ and $\mathsf{O}\neg A$ appear in a deduction. The systems DPI and $SDPI$ also do not trivialize, unless A is a deontically consistent formula.

We have seen, then, how the LDIs can deal with moral dilemmas without trivializing the system. That, however, is not enough to show that they offer a good account of moral dilemmas. We criticized Coniglio and Peron's solution to Chisholm's paradox because it lacked a proper philosophical account backing it up. Likewise, what we have now is a formal treatment of moral dilemmas without an appropriate philosophical interpretation. In the next section, we will provide such a interpretation.

4 Interpreting the formalism

In the previous section, we presented moral dilemmas and showed how the LDIs can model them without succumbing to trivialization. What is still lacking is a philosophical treatment of moral dilemmas within these systems. What we will do in this section is to consider the philosophical merits of how each of the systems discussed so far deals with moral dilemmas. We argue that $DmbC$, $SDmbC$, $SDPI$ and $BDmbC$ do not account for moral dilemmas in a philosophically satisfactory way, suggesting that, from all the systems analyzed, DPI is the most philosophically interesting to deal with moral dilemmas. Besides, we offer a philosophical interpretation of this system regarding its modeling of moral dilemmas. Moreover, we will discuss how this system fits in the philosophical debate around moral dilemmas.

Beginning with $BDmbC$, one can notice that it appeals to two types of obligation to avoid trivialization. Their definitions are based on which modal version of explosion axiom satisfies. Hence, an obligation is weak if it satisfies **(O-E)**°, and strong if it satisfies **(O-E)**. As it was the case when analyzing Chisholm's Paradox, this distinction does not provide for a good solution. Accounting that these are interrelatable by **(BA)** makes the case for the solution using $BDmbC$ even weaker. Even if we have more than one type of obligation at our disposal, in the context of a moral dilemma, we are dealing with two obligations in the strongest sense possible. This would restrict us to analyzing moral dilemmas only when both obligations are of the strong kind, which results in explosion. Therefore, having more than one obligation operator does not suffice to model moral dilemmas. The system $SDmbC$, having only the strong obligation, suffers from the same problems as $BDmbC$.

$DmbC$, on the other hand, relies heavily on the notion of consistency to have explosion, while also having only one obligation. This seems promising at first. However, when we try to justify the reasons why a situation is deontically consistent in this system, we come to an issue. The notion of deontic consistency in $DmbC$ is derived from the notion of consistency defined for propositional formulas and reads $\mathrm{O}{\circ}A$. Since we read O as a moral obligation in this paper, we should, in principle, also give a clear interpretation of $\circ$ that would capture in itself a notion of deontic consistency when paired with O. However, the usual interpretations of such an operator are given in completely different contexts. One interpretation for these paraconsistent logics is epistemic[23], saying that information A is conclusively established as true (or false) (Carnielli and Rodrigues, 2019, p. 3805). Taking such an interpretation, when reading $\mathrm{O}{\circ}A$ we get "it ought morally to be the case that A is conclusively established as being true (false)", which does not sound like a reasonable moral principle to have[24].

[23] We follow (Carnielli and Rodrigues, 2019).
[24] Such an interpretation is also at odds with the community understanding of how paraconsistent logics should be treated. Notably, the paper by Walter Carnielli and Abilio Rodrigues sparked a discussion about interpretations of paraconsistent logics in the community, as shown in (Barrio, 2018), (Barrio

Because of this problematic relation between obligation and consistency, we think that $DmbC$ is ill-fitted to model moral dilemmas.[25]

$SDPI$'s and DPI's formal treatments of dilemmas are similar, but they have a fundamental difference regarding their consistency operator. While $SDPI$ readily trivializes in the propositional level, to which in our reading translates to proving that every propositional formula is provable or true, DPI restricts such an explosion to the modal level. Thus, an agent arriving at a situation where conflict of obligations arises and subsequent deontic trivialization follows cannot infer that *everything follows*. Rather, the only result is that their moral standards are blown out of the window and need to be revised. We understand that if we want a logic to be called paraconsistent *deontic* logic and we want some consistency notion to be defined within such a theory, then the consistency operator must be primitively defined in the deontic context, instead of being derived in the deontic discourse from the propositional case. It means that if we choose to follow the development of the *LFI*s and apply it to the deontic context, then we must have a symbol for *deontic consistency* which is defined formally in analogy to $\circ$, but differs significantly from *consistency simpliciter* when being interpreted.

For these reasons, we think that the system DPI is the best candidate to model moral dilemmas, given the set of systems analyzed here. It refrains from presenting a symbol for consistency *simpliciter* and assumes that such a symbol should appear in the modal level and be constrained to it.

As far as formal definition goes for such an operator, in this work we follow (Coniglio and Peron, 2009) and define it as in DPI, that is, being the operator $\boxminus$ and its behavior being described only by **(Dbc1)**. We thus interpret $\boxminus\phi$ as representing

and Da Re, 2018), (Guercio and Szmuc, 2018), (Arenhart, 2021), (Rodrigues and Carnielli, 2022), and, to some extent, this work.

[25] A way to save $DmbC$ is to interpret $\circ$ as denoting moral consistency. But then, we have to also adapt the logics and thus, it would be a new system altogether, where **(bc1)** is rejected in favor of something closer to **(Dbc1)**, where $\boxminus\alpha$ is now defined as $\circ\alpha$. Now the problem is that **(O-E)** makes no sense and we should also get rid of it. But this results in DPI.

that the proposition ϕ describes an action that occurs in a non-dilemmatic situation. Non-dilemmatic situations, in opposition to dilemmatic situations, are those where contradictory obligations cannot be obtained. Hence, $\boxminus\phi$ represents not only the deontically consistency status of ϕ. It also gives us information about the kind of situation we are dealing with. It is this further information that determines the status of ϕ of being deontically consistent. This interpretation is aligned with the phenomenological approach to moral dilemmas and brings about a formalization that better describes such ideas. The information that a situation is dilemmatic is obtained by the agent through her experience of the situation as one of inescapable moral error.[26] It is, hence, something different from the mere recognition of an inconsistency between duties.

Let us return to Sophie's Choice to see the interpreted system working. We have $\mathrm{O}D$ and $\mathrm{O}\neg D$, Sophie ought to save her daughter and ought to not save her daughter. In our system, that leads to deontic trivialization only under the assumption that $\boxminus D$ holds. But Sophie's situation *is* a dilemmatic one and hence $\boxminus D$ does not obtain. Therefore, we do not get deontic trivialization from moral dilemmas in our system. That means that moral dilemmas do not lead to the trivialization of morality.

This system keeps the inconsistency at the modal level. However, the inconsistency can go beyond it depending on how one decides to extend the system. If one makes a bimodal system with Kant's law, for instance, one can derive $\Diamond(\phi \wedge \psi) \wedge \neg\Diamond(\phi \wedge \psi)$ from the bimodal definition of moral dilemmas. This shows that, if one accepts Kant's Law, moral dilemmas are related to inconsistencies beyond the deontic level. DPI does not trivialize with this kind of inconsistency, so there is no logical problem with it. There is, however, a philosophical question regarding how to understand this kind of inconsistency. As we see it, if there really are moral dilemmas and we can derive $\gamma \wedge \neg\gamma$ for some γ, we are dealing with a real contradic-

[26] For a discussion of the phenomenology of moral dilemmas, see Williams (1965, p. 109ss).

tion. If we are realists about moral dilemmas, we take dilemmas to not be a mere epistemic problem. Hence, we cannot interpret the inconsistencies arising from them using an epistemic or evidential interpretation of paraconsistent inconsistencies. In other words, the contradiction obtained through a conflict of obligations is a true contradiction found in reality. If so, accepting moral dilemmas together with DPI and Kant's Law, for example, leads one to accept dialetheism, the thesis that there are true contradictions. In this aspect, then, we are in agreement with a tradition of philosophers that tend to accept dialetheism because of moral dilemmas (Bohse, 2005; Weber, 2007; Ribeiro Teles, 2021).

If still, someone wants their system to not abide by the existence of moral dilemmas, one can proceed as with *LFI*s, and simply assume every situation is non-dilemmatic. Although the system that results is not as strong as SDL, it should behave closer to classicality. That is not only a desirable formal property of the system, but it also makes our system able to express the disagreement between realists and anti-realists about moral dilemmas. We get deontic consistency back when we accept anti-realism about dilemmas. If $\boxminus\phi$ holds for all ϕ, all situations are non-dilemmatic, i.e., there are no moral dilemmas.

In this section, we carried out a philosophical examination of the formal treatments of moral dilemmas seen in the previous section. We argued that only DPI provided a philosophically adequate modeling of moral dilemmas. We, then, passed to the task of interpreting this modeling. We interpreted $\boxminus\phi$ as representing that the proposition ϕ describes an action that occurs in a non-dilemmatic situation. This straightforward interpretation is motivated by considerations present in the philosophical literature around moral dilemmas. We also discussed how realism coupled with this system seems to be better understood as leading to dialetheism. Finally, we pointed out that DPI has also satisfied the desideratum of being neutral regarding the debate between realists and anti-realists with respect to moral dilemmas, being able to accommodate both stances.

Conclusion

Our main objective in this article was to put paraconsistent deontic logics to good use. We had interesting formal systems proposed by Coniglio and Peron, which developed interesting ideas earlier presented by da Costa and Carnielli. However, the systems were aimed at deontic paradoxes that they were unable to properly deal with. We showed that when talking about Coniglio and Peron's discussion of Chisholm's paradox. The systems solve the formal problem with the paradox, but in a philosophically unmotivated way. We presented the deontic problems related to moral dilemmas and showed how the systems discussed could model them without trivialization. We analyzed the philosophical merits of each system's treatment of moral dilemmas. From our analysis, the system DPI has shown to provide a satisfactory way of formalizing moral dilemmas, while explicitly restraining explosion to the deontic level, which is in accordance with the philosophical robustness we sought after. We also remark that the formal and philosophical developments of ideas here presented shall be the aim of future work, by means of exploring other paraconsistent logics and also extending DPI to include alethic modality.

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References

ANDERSON, A. R.; MOORE, O. K. The Formal Analysis of Normative Concepts. **American Sociological Review**, [American Sociological Association, Sage Publications, Inc.], v. 22, n. 1, p. 9–17, 1957. ISSN 00031224.

ÅQVIST, L. Deontic Logic. In: **Handbook of Philosophical Logic: Volume 8**. Ed. by D. M. Gabbay and F. Guenthner. Dordrecht: Springer Netherlands, 2002. P. 147–264. ISBN 978-94-010-0387-2. DOI: 10.1007/978-94-010-0387-2_3.

ÅQVIST, L. Good Samaritans, Contrary-to-Duty Imperatives, and Epistemic Obligations. **Noûs**, Wiley-Blackwell, v. 1, n. 4, p. 361–379, 1967. DOI: 10.2307/2214624.

ARENHART, J. R. B. The evidence approach to paraconsistency versus the paraconsistent approach to evidence. **Synthese**, Springer Nature, v. 198, n. 12, pp. 11537–11559, 2021. ISSN 00397857, 15730964.

BARRIO, E.; DA RE, B. Paraconsistency and its Philosophical Interpretations. **Australasian Journal of Logic**, Victoria University of Wellington, v. 15, n. 2, p. 151–170, 2018. DOI: 10.26686/ajl.v15i2.4860.

BARRIO, E. A. Models & Proofs: Lfis Without a Canonical Interpretations. **Principia: An International Journal of Epistemology**, Universidade Federal de Santa Catarina, v. 22, n. 1, p. 87–112, 2018. DOI: 10.5007/1808-1711.2018v22n1p87.

BATENS, D. A completeness-proof method for extensions of the implicational fragment of the propositional calculus. **Notre Dame Journal of Formal Logic**, Duke University Press, v. 21, n. 3, p. 509–517, 1980. DOI: 10.1305/ndjfl/1093883174.

BATENS, D. Paraconsistent extensional propositional logics. **Logique et Analyse**, Peeters Publishers, v. 23, n. 90/91, p. 195–234, 1980. ISSN 00245836, 22955836.

BEIRLAEN, M.; STRASSER, C. A Paraconsistent Multi-agent Framework for Dealing with Normative Conflicts. In: LEITE, J. et al. (Eds.). **Computational Logic in Multi-Agent Systems**. Berlin, Heidelberg: Springer Berlin Heidelberg, 2011. P. 312–329. ISBN 978-3-642-22359-4.

BOHSE, H. A Paraconsistent Solution to the Problem of Moral Dilemmas. **South African Journal of Philosophy**, The Philosophical Society of Southern Africa, v. 24, n. 2, p. 77–86, 2005. DOI: 10.4314/sajpem.v24i2.31415.

CARNIELLI, W.; CONIGLIO, M. E. **Paraconsistent Logic: Consistency, Contradiction and Negation**. 1. ed. [S.l.]: Springer Cham, 2016. P. 398. (Logic, Epistemology, and the Unity of Science). DOI: 10.1007/978-3-319-33205-5.

CARNIELLI, W.; CONIGLIO, M. E.; MARCOS, J. Logics of Formal Inconsistency. In: GABBAY, D.; GUENTHNER, F. (Eds.). **Handbook of Philosophical Logic**. Dordrecht: Springer Netherlands, 2007. P. 1–93. ISBN 978-1-4020-6324-4. DOI: 10.1007/978-1-4020-6324-4_1.

CARNIELLI, W.; RODRIGUES, A. An epistemic approach to paraconsistency: a logic of evidence and truth. **Synthese**, v. 196, n. 6, p. 3789–3813, 2019. DOI: 10.1007/s11229-017-1621-7.

CHELLAS, B. F. Basic Conditional Logic. **Journal of Philosophical Logic**, Springer, v. 4, n. 2, p. 133–153, 1975. ISSN 00223611, 15730433.

CHELLAS, B. F. Conditional logic. In: MODAL Logic: An Introduction. [S.l.]: Cambridge University Press, 1980. P. 268–276.

CHISHOLM, R. M. Contrary-To-Duty Imperatives and Deontic Logic. **Analysis**, v. 24, n. 2, p. 33–36, Dec. 1963. ISSN 0003-2638. DOI: 10.1093/analys/24.2.33. eprint: https://academic.oup.com/analysis/article-pdf/24/2/33/478528/24-2-33.pdf.

CONEE, E. Against Moral Dilemmas. **Philosophical Review**, Duke University Press, v. 91, n. 1, p. 87–97, 1982. DOI: 10.2307/2184670.

CONIGLIO, M. E. Logics of Deontic Inconsistency. **Revista Brasileira de Filosofia**, v. 233, p. 162–186, 2009.

CONIGLIO, M. E.; PERON, N. M. A Paraconsistentist Approach to Chisholm's Paradox. **Principia: An International Journal of Epistemology**, Universidade Federal de Santa Catarina, v. 13, n. 3, p. 299–326, 2009. DOI: 10.5007/1808-1711.2009v13n3p299.

COSTA, N. C. A. da; CARNIELLI, W. A. On Paraconsistent Deontic Logic. **Philosophia**, v. 16, n. 3-4, p. 293–305, 1986. DOI: 10.1007/bf02379748.

CRUZ, Â. M. P. **Lógica deôntica paraconsistente**: paradoxos e dilemas. [S.l.]: EDUFRN, 2005. P. 96. (Coleção Lógica). ISBN ISBN 85-7316-345-3.

FORRESTER, J. W. Gentle Murder, or the Adverbial Samaritan. **Journal of Philosophy**, Journal of Philosophy Inc, v. 81, n. 4, p. 193–197, 1984. DOI: 10.2307/2026120.

GUERCIO, N. L.; SZMUC, D. Remarks on the Epistemic Interpretation of Paraconsistent Logic. **Principia: An International Journal of Epistemology**, Universidade Federal de Santa Catarina, v. 22, n. 1, p. 153–170, 2018. DOI: 10.5007/1808-1711.2018v22n1p153.

HILPINEN, R.; MCNAMARA, P. Deontic Logic: A historical survey and introduction. In: GABBAY, D.; HORTY, J.; PARENT, X. (Eds.). **Handbook of Deontic Logic and Normative Systems**. [S.l.]: College Publications, 2013. P. 3–137. ISBN 9781848901285.

HINTIKKA, J. Some Main Problems of Deontic Logic. In: **Deontic Logic: Introductory and Systematic Readings**. Ed. by Risto Hilpinen. Dordrecht: Springer Netherlands, 1971. P. 59–104. ISBN 978-94-010-3146-2. DOI: 10.1007/978-94-010-3146-2_3.

LEMMON, E. J. Deontic Logic and the Logic of Imperatives. **Logique Et Analyse**, v. 8, n. 29, p. 39–61, 1965.

LEWIS, D. Semantic Analyses for Dyadic Deontic Logic. In: **Logical Theory and Semantic Analysis: Essays Dedicated to STIG KANGER on His Fiftieth Birthday**. Ed. by Sören Stenlund. Dordrecht: Springer Netherlands, 1974. P. 1–14. ISBN 978-94-010-2191-3. DOI: 10.1007/978-94-010-2191-3_1.

MARCUS, R. B. Moral Dilemmas and Consistency. **The Journal of Philosophy**, Journal of Philosophy, Inc., v. 77, n. 3, p. 121–136, 1980. ISSN 0022362X.

MCGINNIS, C. **Paraconsistency and deontic logic: Formal systems for reasoning with normative conflicts**. 2007. PhD thesis – University of Minnesota.

MERLUSSI, P. Há Dilemas Morais? **Ethic@ - An International Journal for Moral Philosophy**, Universidade Federal de Santa Catarina, v. 12, n. 2, p. 207–226, 2013. DOI: 10.5007/1677-2954.2013v12n2p207.

MEYER, J.-J.; DIGNUM, F.; WIERINGA, R. **The paradoxes of deontic logic revisited: a computer science perspective**. [S.l.]: Utrecht University, Sept. 1994. (Technical Report, UU-CS-1994-38).

PERON, N. M.; CONIGLIO, M. E. Logics of Deontic Inconsistencies and Paradoxes. **CLE e-prints**, v. 8, n. 6, p. 10, 2008.

PRAKKEN, H.; SERGOT, M. Contrary-to-duty obligations. **Studia Logica**, v. 57, p. 91-115, 1 1996. DOI: 10.1007/BF00370671.

PRAKKEN, H.; SERGOT, M. Dyadic Deontic Logic and Contrary-to-Duty Obligations. In: **Defeasible Deontic Logic**. Ed. by Donald Nute. Dordrecht: Springer Netherlands, 1997. P. 223–262. ISBN 978-94-015-8851-5. DOI: 10.1007/978-94-015-8851-5_10.

PRIOR, A. N. Escapism: the logical basis of Ethics. In: **Essays on Moral Philosophy**. Ed. by Abraham Irving Melden. [S.l.]: University of Washington Press, 1958. chap. 6, p. 135–146.

RIBEIRO TELES, E. Uma possível solução para o problema da existência dos dilemas morais. **Polymatheia - Revista de Filosofia**, v. 12, n. 21, p. 48–69, 2021.

RODRIGUES, A.; CARNIELLI, W. On Barrio, Lo Guercio, and Szmuc on Logics of Evidence and Truth. **Logic and Logical Philosophy**, v. 31, n. 2, p. 313–338, 2022. DOI: 10.12775/LLP.2022.009.

ROSS, A. Imperatives and Logic. **Philosophy of Science**, [Cambridge University Press, The University of Chicago Press, Philosophy of Science Association], v. 11, n. 1, p. 30–46, 1944. ISSN 00318248, 1539767X.

ROSS, W. D. **The Right and the Good.Some Problems in Ethics**. [S.l.]: Clarendon Press, 1930.

VAN FRAASSEN, B. C. Values and the Heart's Command. **Journal of Philosophy**, Journal of Philosophy Inc, v. 70, n. 1, p. 5–19, 1973. DOI: 10.2307/2024762.

WEBER, Z. On Paraconsistent Ethics. **South African Journal of Philosophy**, The Philosophical Society of Southern Africa, v. 26, n. 2, p. 239–244, 2007. DOI: 10.4314/sajpem.v26i2.31477.

WILLIAMS, B. A. O. Symposium: Ethical Consistency. **Aristotelian Society Supplementary Volume**, Wiley-Blackwell, v. 39, p. 103–124, 1965.

WRIGHT, G. H. von. Deontic Logic. **Mind**, [Oxford University Press, Mind Association], v. 60, n. 237, p. 1–15, 1951. ISSN 00264423, 14602113.

ZIMMERMAN, M. J. Remote Obligation. **American Philosophical Quarterly**, University of Illinois Press, v. 24, n. 2, p. 199–205, 1987.

A Completeness Proof for *DPI*

Definition A.1. The logic PI is the logic over the signature $\Sigma = \{\to, \wedge, \vee, \neg\}$ given by the following axioms[27]:

1. $(p \to (q \to p))$
2. $((p \to q) \to p) \to p$
3. $(p \to q) \to ((q \to r) \to (p \to r))$
4. $(p \wedge q) \to p$
5. $(p \wedge q) \to q$
6. $p \to (q \to (p \wedge q))$
7. $p \to (p \vee q)$
8. $q \to (p \vee q)$
9. $(p \to r) \to ((q \to r) \to ((p \vee q) \to r))$
10. $p \vee \neg p$

and the rules of Modus ponens and Uniform Substitution.

From PI, we can construct a modal system that satisfies the axiom for $\boxminus$ as presented in the body of the text.

Definition A.2. Let $\Sigma^* = \Sigma \cup \{O, \boxminus\}$ be a signature and $For(\Sigma^*)$ the set of formulas over Σ^*. The logic DPI is the modal extension of PI given by the following axioms:

(K) $O(\alpha \to \beta) \to (O\alpha \to O\beta)$

(Dbc1) $\boxminus\alpha \to (O\alpha \to (O\neg\alpha \to O\beta))$ for all $\beta \in For(\Sigma^*)$

(O-Nec) If $\vdash \alpha$, then $\vdash O\alpha$

Definition A.3. Let $\mathcal{A}_3 := \{A, \tilde{\wedge}, \tilde{\vee}, \tilde{\neg}, \tilde{\to}, \tilde{O}, \tilde{\boxminus}\}$ be a multialgebra with domain $A = \{T, t, F\}$, where $T := (1,0)$, $t := (1,1)$ and $F := (0,1)$. Let $\mathcal{D} = \{T, t\}$ denote the designated truth values and define $\mathcal{M}_3 := \langle \mathcal{A}_3, \mathcal{D} \rangle$ to be an Nmatrix over signature Σ^*. Moreover, let W be a nonempty set of worlds, $R \subseteq W^2$ be an accessibility relation and $\{v_w\}_{w \in W}$ a family of valuations.

[27] The techniques presented here were developed in a collaboration between prof. Marcelo Coniglio and Mahan Vaz and is more thorouhly presented in a currently unpublished draft being in its final stages of development. There, the technique was applied to various systems other than DPI.

Definition A.4. We call $\mathscr{B}$ a swap structure for DPI over $\mathscr{A}_3$ if its domain is $\mathscr{B}_{\mathscr{A}_3} = \{(c_1, c_2) \in A : c_1 \sqcup c_2 = 1\}$[28]

Definition A.5. Let $A = \mathscr{B}_{\mathscr{A}_3}$ be the domains of swap structures for DPI and assume $a, b \in A$. We define the multioperations $\tilde{\wedge}, \tilde{\vee}, \tilde{\to}, \tilde{\neg}$, a special case multioperator $\tilde{\mathrm{O}}$:

1. $a \tilde{\wedge} b := \{(c_1, c_2) \in A : c_1 = a_1 \sqcap b_1\}$
2. $a \tilde{\vee} b := \{(c_1, c_2) \in A : c_1 = a_1 \sqcup b_1\}$
3. $a \tilde{\to} b := \{(c_1, c_2) \in A : c_1 = a_1 \supset b_1\}$
4. $\tilde{\neg} a := \{(c_1, c_2) \in A : c_1 = a_2\}$
5. $\tilde{\mathrm{O}}(X) := \{(c_1, c_2) \in A : c_1 = \bigsqcap\{x_1 : x \in X\}\}$ for $X \neq \varnothing$ such that $X \subseteq \mathscr{B}_{\mathscr{A}_3} = A$

In addition to the above multioperators, we can define a multioperator $\tilde{\boxminus}$ as shown below:

$$\tilde{\boxminus} X := \{(c_1, c_2) \in A : c_1 \leq \sim(\bigsqcap\{x_1 : x \in X\} \sqcap \bigsqcap\{x_2 : x \in X\})\}$$

for $X \neq \varnothing$ such that $X \subseteq \mathscr{B}_{\mathscr{A}_3} = A$

Definition A.6. We let W be a set of sets. For each $w, w' \in W$ a function $v_w : For_\Sigma \to \mathscr{M}_3$ is a *swap valuation* for DPI if every condition below is satisfied:

1. $v_w(p) \in \mathscr{A}_3$

2. $v_w(\alpha \wedge \beta) \in v_w(\alpha) \tilde{\wedge} v_w(\beta)$

3. $v_w(\alpha \vee \beta) \in v_w(\alpha) \tilde{\vee} v_w(\beta)$

4. $v_w(\alpha \to \beta) \in v_w(\alpha) \tilde{\to} v_w(\beta)$

5. $v_w(\neg\alpha) \in \tilde{\neg} v_w(\alpha)$

6. $v_w(\mathrm{O}\alpha) \in \tilde{\mathrm{O}}\{v_{w'}(\alpha) : wRw'\}$

7. $v_w(\boxminus\alpha) \in \tilde{\boxminus}\{v_{w'}(\alpha) : wRw'\}$

Definition A.7. We call $\mathscr{M} = \langle W, R, \{v_w\}_{w \in W}\rangle$ a swap Kripke model for the logic DPI.

Remark 1. Let $i \in \{1, 2\}$ and $w \in W$. We define $\pi_i(v_w(\alpha))$ to be the projection of the tuple $v_w(\alpha)$ on its i-th coordinate. For the sake of simplicity, we adopt the notation $\alpha_{(i,w)}$ to denote $\pi_i(v_w(\alpha))$.

Lemma A.8. Let $w \in W$ and $\alpha, \beta \in For(\Sigma^*)$. Moreover, let v_w be as in A.6. Then

[28] The symbols $\sqcup, \sqcap, \supset, \sim$ respectively refer to the join, meet, implication and complement operations in a Boolean algebra. We use them to refer to the operations in each coordinate of the tuple, which behave as in a Boolean algebra.

1. $(\alpha \wedge \beta)_{(1,w)} = (\alpha_{(1,w)} \sqcap \beta_{(1,w)})$

2. $(\alpha \vee \beta)_{(1,w)} = (\alpha_{(1,w)} \sqcup \beta_{(1,w)})$

3. $(\alpha \to \beta)_{(1,w)} = (\alpha_{(1,w)} \supset \beta_{(1,w)})$

4. $(\neg\alpha)_{(1,w)} = \alpha_{(2,w)}$

5. $(\mathrm{O}\alpha)_{(1,w)} = \bigsqcap\{\alpha_{(1,w')} : wRw'\}$

We can also define a similar behavior for $\boxminus$:

6. $(\boxminus\alpha)_{(1,w)} \leq \sim((\mathrm{O}\alpha)_{(1,w)} \sqcap (\mathrm{O}(\neg\alpha))_{(1,w)})$

Proof. Items 1. through 4. are immediate from the definitions above. For item 5., consider $\tilde{\mathrm{O}}\{v_{w'}(\alpha) : wRw'\}$. By definition A.5,

$$\tilde{\mathrm{O}}\{v_{w'}(\alpha) : wRw'\} = \{c \in A : c_1 = \bigsqcap\{x_{(1,w')} : x \in \{v_{w'}(\alpha) : wRw'\}\}\}$$

thus our result follows by item 6. of definition A.6.

The case for item 6. follows from items 4. and 5. and the Boolean complement. □

Proposition A.9. Let $a \in \mathcal{B}_{\mathcal{A}_3}$. Then for any $w \in W$ and for any $\alpha \in For(\Sigma)$, $v_w(\alpha) \in \mathcal{D}$ if and only if $\alpha_{(1,w)} = 1$.

Proof. This is immediate from definition A.3 and the previous lemma. □

Definition A.10. Let $\mathcal{M} = \langle W, R, \{v_w\}_{w\in W}\rangle$ be as in definition A.6. Then for any formula $\alpha \in For(\Sigma^*)$, we say that α is $\mathcal{M}_v$-true in a world w, denoted $\mathcal{M}, w \Vdash \alpha$, if $v_w(\alpha) \in \mathcal{D}$. If $\Gamma \subseteq For(\Sigma^*)$, then $\mathcal{M}, w \Vdash \Gamma$ denotes $\mathcal{M}, w \Vdash \gamma$ for all $\gamma \in \Gamma$.

Definition A.11. Let $\Gamma \subseteq For(\Sigma^*)$. Then we say that α is a logical consequence of Γ, denoted $\Gamma \vDash \alpha$, if for all $\mathcal{M}$, if, for all $w \in W$ $\mathcal{M}, w \Vdash \Gamma$, then $\mathcal{M}, w \Vdash \alpha$.

Proposition A.12. The system DPI does not validate axiom **(DD)**.

Proof. Let $w \in W$. Take $v_w(\mathrm{O}\alpha) = v_w(\mathrm{O}\neg\alpha) = (1, 0)$, thus $v_w(\neg\mathrm{O}\neg\alpha) = (0, 1)$. This is satisfiable when for every $w' \in W$ such that wRw' we have $v_{w'}(\alpha) = (1, 1)$. □

Theorem A.13 (Soundness). For every $\Delta \cup \{\varphi\} \subset For(\Sigma^*)$,

$$\text{if } \Delta \vdash_{DPI} \varphi, \text{ then } \Delta \vDash \varphi$$

Proof. The result for the PI axioms follows from lemma A.8.

For modus ponens, it is immediate to see that it satisfies the criteria. For Necessitation, suppose α is a theorem. Then for every $w \in W$, $\alpha_{(1,w)} = 1$. In particular, it is the case for every $w' \in W$ such that wRw', from which follows that $(\mathrm{O}\alpha)_{(1,w)} = 1$.

For (O-K), assume $(\mathrm{O}(\alpha \to \beta))_{(1,w)} = 1$ and that $(\mathrm{O}(\alpha))_{(1,w)} = 1$. We then have that $(\alpha \to \beta)_{(1,w')} = 1$ and that $\alpha_{(1,w')} = 1$ for every $w' \in W$ such that wRw'. Hence, it follows that $\beta_{(1,w')} = 1$ for every w' such that wRw'.

For (Dbc1), let $(\boxminus\alpha)_{(1,w)} = (\mathrm{O}\alpha)_{(1,w)} = (\mathrm{O}\neg\alpha)_{(1,w)} = 1$. But then $\sim((\mathrm{O}\alpha)_{(1,w)} \sqcap (\mathrm{O}\neg\alpha)_{(1,w)}) = 0$, so

$$1 = (\boxminus\alpha)_{(1,w)} > \sim((\mathrm{O}\alpha)_{(1,w)} \sqcap (\mathrm{O}\neg\alpha)_{(1,w)}) = 0$$

which contradicts item 4. of lemma A.8.

(O-Nec) is proven by noticing that if $\alpha_{(1,w)} = 1$ for all $w \in W$, then taking $(\mathrm{O}\alpha)_{(1,w')} = 0$ for some $w' \in W$ means that there is w'' such that $w'Rw''$ and $\alpha_{(1,w'')} = 0$, which contradicts our assumption.

□

In order to prove completeness for DPI, we build canonical models based upon our swap structures. We use the method of ψ-saturation for construction of maximal consistent sets, together with the denecessitation for the accessibility relation. This will permit us to create the models for which completeness results will hold.

Definition A.14. Let $W_{can} = \{\Delta \subseteq For(\Sigma^*) : \Delta$ is a ψ-saturated in $DPI\}$, that is, we say $\Delta \in W$ is ψ-saturated if $\Delta \nvdash \psi$ and if $\varphi \notin \Delta$, then $\Delta \cup \{\varphi\} \vdash \psi$, for some $\psi \in For(\Sigma^*)$.

Definition A.15. $Den(\Delta) := \{\varphi \in For(\Sigma^*) : \mathrm{O}\varphi \in \Delta\}$

Definition A.16. Let $R_{can} \subseteq W \times W$ be given by:

$$\Delta R_{can}\Theta \text{ if } Den(\Delta) \subseteq \Theta$$

for $\Delta, \Theta \in W$

Definition A.17. For each $\Delta \in W_{can}$ let $\alpha_{(1,\Delta)} = 1$ iff $\alpha \in \Delta$

Lemma A.18. For any $\Delta \in W_{can}$, the following holds:

1. $\alpha \wedge \beta \in \Delta$ iff $\alpha, \beta \in \Delta$

2. $\alpha \vee \beta \in \Delta$ iff $\alpha \in \Delta$ or $\beta \in \Delta$

3. $\alpha \rightarrow \beta \in \Delta$ iff $\alpha \notin \Delta$ or $\beta \in \Delta$

4. If $\neg\alpha \notin \Delta$, then $\alpha \in \Delta$

5. $\mathrm{O}\alpha \in \Delta$ iff $\alpha \in \Delta'$ for all $\Delta' \in W$ such that $\Delta R_{can}\Delta'$

6. If $\mathrm{O}\alpha \in \Delta$ and $\mathrm{O}\neg\alpha \in \Delta$ then $\boxminus\alpha \notin \Delta$

Proof. Items 1.through 4. and the left-to-right direction of item 5. are immediate from the definitions and the fact that Δ is φ-saturated.

To prove the right-to-left direction for 5., assume that $\mathrm{O}\alpha \notin \Delta$ and suppose, for a contradiction, that $Den(\Delta) \vdash \alpha$. Thus, there are $\beta_1, \ , \beta_n \in Den(\Delta)$ such that

$$\bigwedge_{i=1}^{n} \beta_i \vdash \alpha$$

For simplicity, we denote $\bigwedge_{i=1}^{n} \beta_i$ by B and notice that $\mathsf{O}B \in \Delta$. Hence, we have

$$B \vdash \alpha$$

Since PI has the deduction metatheorem (Batens, 1980a), we have:

$$\vdash B \to \alpha$$

And applying necessitation and (K),

$$\vdash \mathsf{O}B \to \mathsf{O}\alpha$$

Using the deduction metatheorem once more we get

$$\mathsf{O}B \vdash \mathsf{O}\alpha$$

and since $\mathsf{O}B \in \Delta$ by hypothesis,

$$\Delta \vdash \mathsf{O}\alpha$$

which contradicts our initial assumption. Since $Den(\Delta) \nvdash \alpha$, there is some Δ' such that $Den(\Delta) \subseteq \Delta'$ and Δ' is α-saturated. Therefore, there is Δ' such that $\Delta R_{can} \Delta'$ and $\alpha \notin \Delta'$.

To prove item 6. Assume $\mathsf{O}\alpha \in \Delta$ and $\mathsf{O}\neg\alpha \in \Delta$. By item 5., for each $\Delta' \in W$ such that $\Delta R_{can} \Delta'$, $\alpha \in \Delta'$ and $\neg\alpha \in \Delta'$., i.e., $v_{\Delta'}(\alpha) = t$, for all Δ' such that $\Delta R_{can} \Delta'$. But this means that

$$0 = {\sim}(\bigsqcap\{\alpha_{(1,\Delta')} : \Delta R \Delta'\} \sqcap \bigsqcap\{\alpha_{(2,\Delta')} : \Delta R \Delta'\}) = (\boxminus\alpha)_{(1,\Delta)}$$

hence $\boxminus\alpha \notin \Delta$. □

Proposition A.19. For any $\Delta \in W$, $v_\Delta(\varphi) \in \mathcal{D}$ if and only if $\varphi \in \Delta$.

Proof. It is an immediate consequence from proposition A.9 and definition A.17. □

Proposition A.20 (Completeness). For any set $\Gamma \cup \{\varphi\} \subseteq For_\Sigma$, If $\Gamma \vDash \varphi$, then $\Gamma \vdash \varphi$

Proof. Suppose, to the contrary, that $\Gamma \nvdash \varphi$. Thus, there is some φ-saturated $\Delta \in W$ such that $\Gamma \subseteq \Delta$. Since $\Delta \nvdash \varphi$, then $\varphi \notin \Delta$. Assume that $\mathcal{M}$ is the canonical swap Kripke model for DPI. This means that $\mathcal{M}, \Delta \Vdash \Gamma$, since $\Gamma \subseteq \Delta$, but $\mathcal{M}, \Delta \nVdash \varphi$. □